The Dynamic Behavior of Consumption under Uncertainty

The Dynamic Behavior of Consumption and Portfolio Choice under Uncertainty
Optimization of Discounted Utility of a Representative Household

$$\max U = B \int_{t=0}^{\infty} e^{-\beta t} \frac{c(t)^{1-\theta}}{1-\theta} dt$$

under the constraint

$$k(t) = r(t)k(t) + w(t) - c(t) - (n + g)k(t)$$

where \( \beta = \rho - n - (1-\theta)g > 0 \)
The First Order Conditions for the Maximization of the Intertemporal Utility of the Representative Household

\[ c(t) = \frac{1}{\theta} (r(t) - \rho - \theta g) c(t) \]

\[ k(t) = r(t) k(t) + w(t) - c(t) - (n + g) k(t) \]

The first equation is the Euler equation for consumption and the second the accumulation equation.
The Intertemporal Budget Constraint of the Representative Household

The accumulation equations is a first order differential equation with variable coefficients. As a result, its solution for every $T \geq 0$ takes the form,

$$
e^{-\left(\int_{t=0}^{T} r(v) dv - (n-g)T\right)} k(T) + \int_{t=0}^{T} e^{-\left(\int_{v=0}^{t} r(v) dv - (n-g)t\right)} c(t) dt = k(0) + \int_{t=0}^{T} e^{-\left(\int_{v=0}^{t} r(v) dv - (n-g)t\right)} w(t) dt$$

This inter-temporal budget constraint implies that at time $0$, the present value of labor income between $0$ and $T$, plus the initial capital stock at $0$, is equal to the present value of consumption between $0$ and $T$, plus the present value of the capital stock at $T$. The term that includes the integral of interest rates is a term that transforms a unit of income, consumption or capital at instant $t$ to its present value at instant $0$. If the interest rate was constant, this term would simplify to $-rt$. 

Prof George Alogoskoufis, Dynamic Macroeconomic Theory, 2015
The Average Interest Rate and the Inter-temporal Budget Constraint

We can define the average interest rate between $0$ and $t$ as,

$$\bar{r}(t) = \frac{1}{t} \int_{v=0}^{t} r(v) dv$$

With this definition, the inter-temporal budget constraint can be written as,

$$e^{-\left(\bar{r}(T) - n - g\right)T} k(T) + \int_{t=0}^{T} e^{-\left(\bar{r}(t) - n - g\right)t} c(t) dt = k(0) + \int_{t=0}^{T} e^{-\left(\bar{r}(t) - n - g\right)t} w(t) dt$$
The Inter-temporal Budget Constraint with an Infinite Time Horizon

As the time horizon $T$ tends to infinity, the inter-temporal budget constraint can be written as,

$$\lim_{T \to \infty} e^{-\left(\tilde{r}(T)-n-g\right)T} k(T) + \int_{t=0}^{\infty} e^{-\left(\tilde{r}(t)-n-g\right)t} c(t)dt = k(0) + \int_{t=0}^{\infty} e^{-\left(\tilde{r}(t)-n-g\right)t} w(t)dt$$

As the time horizon tends to infinity, the term on the left must tend to zero, as the household derives utility only from consumption and not the capital stock. (transversality condition). As a result, the inter-temporal budget constraint with an infinite time horizon takes the form,

$$\int_{t=0}^{\infty} e^{-\left(\tilde{r}(t)-n-g\right)t} c(t)dt = k(0) + \int_{t=0}^{\infty} e^{-\left(\tilde{r}(t)-n-g\right)t} w(t)dt$$
The Transversality Condition and the Time Horizon

If the horizon of the household was $T$, then the optimal capital stock at instant $T$ would be equal to zero. As the time horizon $T$ tends to infinity, this implies that,

$$\lim_{T \to \infty} e^{-\left(\bar{r}(T) - n - g\right)T} k(T) = 0$$

If this condition is not satisfied, for example if the above limit is positive, then the household could along the optimal path increase its inter-temporal utility by consuming a larger part of its capital. If the above limit is negative, then the household would be accumulating unsustainable debts (negative capital) along the optimal path, which is not consistent with its inter-temporal budget constraint. Therefore, the only optimal path consistent with the inter-temporal budget constraint of the representative household is the one which satisfies the condition that the present value of its capital stock tends to zero as time tends to infinity. This is the infinite horizon transversality condition. It is satisfied as long as the capital stock per efficiency unit of labor does not increase (or decrease) at a rate faster than $r-n-g$, which is the same as saying that the aggregate capital stock does not increase (or decrease) at a rate faster than $r$. 

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The Consumption Function of the Representative Household with an Infinite Time Horizon

If we solve (integrate) the differential equation describing the Euler equation for consumption, then we find that consumption at time $t$ is defined by,

$$c(t) = c(0)e^{\frac{1}{\theta}\left(r(t) - \rho - \theta g\right)t}$$

Substituting for consumption $c(t)$ in the inter-temporal budget constraint, the consumption function at instant $0$ takes the form,

$$c(0) = \gamma(0)\left(k(0) + \int_{t=0}^{\infty} e^{-\left(r(t) - n - g\right)t} w(t)dt\right)$$

where

$$\gamma(0) = \left(\int_{t=0}^{\infty} e^{\frac{r(t)(1-\theta) - \rho + \theta n}{\theta}t} dt\right)^{-1}$$
The Interpretation of the Consumption Function of the Representative Household

• The representative household consumes a share of its total wealth $\gamma(0)$, that depends on the evolution of the average real interest rate, the pure rate of time preference rate $\rho$, the elasticity of inter-temporal substitution of consumption $1/\theta$, and the population growth rate $n$.

• The impact of the average real interest rate on the proportion of total wealth that is consumed depends on the elasticity of inter-temporal substitution of consumption $1/\theta$. An increase in average real interest rates has two kinds of effects on the average consumption to total wealth ratio: an inter-temporal substitution effect, and an income effect. First, it induces the household to substitute current for future consumption, and increases the cost of current consumption relative to future consumption. This is the inter-temporal substitution effect in consumption, which tends to decrease current consumption. Second, an increase in interest rates increases income from capital, and tends to increase both current and future consumption. This is the income effect, which tends to increase current consumption.

• If the inter-temporal elasticity of substitution in consumption is greater than one ($\theta < 1$), then consumption as a proportion of total wealth decreases when interest rates rise, because the negative substitution effect is stronger than the positive income effect, and thus prevails on the income effect. If the inter-temporal elasticity of substitution is less than unity ($\theta > 1$), then consumption as a proportion of total wealth increases when interest rates rise, because the positive income effect is stronger than the negative substitution effect. Finally, if $\theta = 1$, which is the case with logarithmic preferences, the two results cancel each other out, and consumption as a proportion of total wealth is independent of the path of real interest rates.
The Consumption Function of the Representative Household with a Constant Real Interest Rate

If the real interest rate is constant at $r$, then the share of total wealth that is consumed is given by,

$$
\gamma(0) = \left( \int_{t=0}^{\infty} e^{\theta \frac{1}{t}(r(1-\theta)-\rho + \theta n)t} \right)^{-1} = \frac{r(1-\theta)-\rho + \theta n}{\theta \lim_{t \to \infty} e^{\theta \frac{1}{t}(r(1-\theta)-\rho + \theta n)t} - \lim_{t \to 0} e^{\theta \frac{1}{t}(r(1-\theta)-\rho + \theta n)t}} = \frac{1}{\theta} \left( \rho - \theta n - r(1-\theta) \right)
$$

For this to be positive, we must have $r < (\rho - \theta n)/(1-\theta)$. On the balanced growth path, $r = \rho + \theta g$. As a result, the share of total wealth that is consumed on the balanced growth path is given by,

$$
\gamma(0) = \left( \rho - n - (1-\theta)g \right)
$$

which we have already assumed is positive in order to have a well defined inter-temporal optimization problem for the representative household.
The Consumption Function of the Representative Household with a Unitary Elasticity of Inter-temporal Substitution of Consumption

The consumption function is considerably simpler if the elasticity of inter-temporal substitution of consumption is equal to unity (logarithmic preferences). In this case,

\[ \gamma(0) = \left( \int_{t=0}^{\infty} e^{-r(t)(1-\theta)-\rho+\theta n} t \, dt \right)^{-1} = \left( \int_{t=0}^{\infty} e^{-(\rho-n)t} \right)^{-1} = \frac{\rho - n}{\lim_{t \to \infty} e^{-(\rho-n)t} + \lim_{t \to 0} e^{-(\rho-n)t}} = \rho - n \]

Given that we have assumed that \( \rho > n \), with a unitary elasticity of inter-temporal substitution of consumption, the share of consumption in total wealth is equal to the difference between the pure rate of time preference and the population growth rate.
Real Interest Rates and Aggregate Consumption in the Representative Household Model

• Finally, it is also worth noting that the overall impact of real interest rates on consumption is not limited to the impact on the propensity to consume out of total wealth.

• An increase in real interest rates leads to a decrease in the present value of future labor income, reducing the overall wealth of the representative household, and leading to a reduction in consumption, even if the inter-temporal elasticity of substitution is equal to one.

• Essentially, the effects of real interest rates on the present value of income from employment, i.e. the wealth effects of real interest rates, reinforce the substitution effect on current consumption.
Consumption under Uncertainty

• We next examine dynamic models of consumer choice under uncertainty. We continue, as in the Ramsey model, to take the decision of the household with regard to labor supply as given, assuming that each household provides a unit of labor per period. We also assume that the household can borrow and lend freely in competitive capital markets.

• The choice of consumption under conditions of uncertainty is linked to the portfolio allocation decisions of the household (Samuelson 1969, Merton 1969).

• Under uncertainty, consumption generally depends on the same factors as under certainty, only in the case of quadratic preferences, which guarantee certainty equivalence. In the case of quadratic preferences we can derive the permanent income model of consumption.

• In all other cases, under uncertainty, we cannot go beyond the first order conditions and solve explicitly for consumption, unless we make further restrictive assumptions about the preferences of households or the variability of labor income.

• With regard to portfolio choice, this model results in the consumption capital asset pricing model. This suggests that, under quadratic preferences, the expected return premium of a risky asset is proportional to the covariance of its return with consumption. This factor of proportionality is sometimes referred to as a consumption beta, and can be used to explain the valuation of risky assets.
The Optimization of Expected Utility under Uncertainty

We assume a household with a time horizon \( T \), which is uncertain with regard to its future labor income and with regard to the future returns on its portfolio of assets.

In period 0 the household maximizes,

\[
E_0 \left( \sum_{t=0}^{T-1} \left( \frac{1}{1+\rho} \right)^t u(C_t) \right)
\]

under the asset accumulation constraint,

\[
A_{t+1} = (A_t + Y_t - C_t) \left[ (1 + r_t)\omega_t + (1 + x_t)(1 - \omega_t) \right]
\]
Definitions

$E_t$ denotes a mathematical expectation based on the set of available information available in period $t$.

$\rho$ is the pure rate of time preference of the household.

$u$ a periodic utility function, depending on consumption.

$A_t$ is the value of the portfolio of assets of the household at the beginning of period $t$.

$Y_t$ is labor income, which is assumed to be a random variable whose value is known in period $t$ but not before.

Gross savings of the household are defined by, $A_t + Y_t - C_t$

The household allocates its gross savings between a “safe” asset, with a certain return $r_t$, and a “risky” asset, with uncertain return $x_t$.

The portfolio allocation decision of the household is determined by the percentage $\omega$ of its assets that is invested in the “safe” asset. The term in brackets thus denotes the average rate of return of the household’s portfolio.
Solving the Problem of the Household using Dynamic Programming

The household chooses a consumption and portfolio allocation plan for period 0, in the knowledge that it will be able to choose a new plan in the following period 1, a new plan in the following period 2, and so on, until the penultimate period $T-1$. The easiest method of solving of dynamic problems under uncertainty is the method of stochastic dynamic programming.

Dynamic programming converts multi-period problems into a sequence of simpler two period selection problems. The first step is the introduction of a value function $V_t(A_t)$, which is defined as,

$$V_t(A_t) = \max E_t \left( \sum_{s=t}^{T-1} \left( \frac{1}{1+\rho} \right)^{s-t} u(C_s) \right)$$

The value function in period $t$ is the discounted present value of the expected utility of the household, calculated under the assumption that the household follows the optimal program of consumption and portfolio allocation. This optimal value depends on the value of the portfolio of the household at the beginning of period $t$, which is the only state variable affecting the household.
The Bellman Equation and the First Order Conditions

The value function satisfies the following recursive equation, which is known as the Bellman equation.

$$V_t(A_t) = \max_{\{C_t, \omega_t\}} \left\{ u(C_t) + \frac{1}{1 + \rho} E_t \left[ V_{t+1}(A_{t+1}) \right] \right\}$$

The value function in period $t$ is equal to the maximum utility of consumption in period $t$ plus the discounted expected value function in period $t+1$. The first order conditions for the maximization of the value function under the asset accumulation constraint are,

$$u'(C_t) = E_t \left[ \frac{1}{1 + \rho} \left( (1 + r_t)\omega_t + (1 + x_t)(1 - \omega_t) \right) V_{t+1}'(A_{t+1}) \right]$$

$$E_t \left[ V_{t+1}'(A_{t+1})(r_t - x_t) \right] = 0$$
Interpretation of the First Order Conditions

Applying the envelope theorem to the Bellman Equation, i.e. the effects of a small change in the value of the portfolio of assets $A_t$ on both of its sides, we get that the marginal value of the household portfolio of assets is equal to the marginal utility of consumption.

$$V_t'(A_t) = u'(C_t)$$

After some substitutions in the first order conditions, we get that,

$$u'(C_t) = \frac{1 + r_t}{1 + \rho} E_t[u'(C_{t+1})]$$

These conditions have a simple interpretation, which is a generalization of the interpretation of the Euler equation for consumption in the Ramsey problem. Recall, that the Euler equation for consumption in the Ramsey problem suggests that the marginal rate of substitution between the levels of consumption in the two periods must be equal to the marginal rate of transformation for both the “safe” and the “risky” asset.
Implications of the First Order Conditions for the Dynamic Evolution of Consumption

The first order conditions imply strong restrictions for the dynamic behavior of consumption. The first condition implies that,

\[
\frac{1 + r_t}{1 + \rho} u'(C_{t+1}) = u'(C_t) + \varepsilon_{t+1} \quad \text{where,} \quad E_t(\varepsilon_{t+1}) = 0
\]

Given the marginal utility of consumption \( u'(C_t) \), there no additional information available in period \( t \) that could help predict \( u'(C_{t+1}) \), the future marginal utility of consumption.

Assuming that the utility function is quadratic in consumption, and that the rate of return of the “safe” asset is equal to the pure rate of time preference, then this condition takes the form,

\[
C_{t+1} = C_t + \varepsilon_{t+1}
\]

Consumption follows a “random walk”. Given the level of consumption in period \( t \), no other variable known in period \( t \) can help predict consumption in period \( t+1 \). This prediction of the model was first highlighted by Hall (1978), who investigated it empirically. This has generated a host of theoretical and empirical follow up studies of this prediction.
The Consumption Capital Asset Pricing Model (CAPM)

The first order conditions can also be used to determine the rate of return of the risk free asset and the expected rate of return, and hence the price of the risky asset. This requires that all individual households are alike, i.e. that there is a representative household.

From the Euler equation for the risk free asset, its rate of return will satisfy,

$$1 + r_t = (1 + \rho) \frac{u'(C_t)}{E_t[u'(C_{t+1})]}$$

From the Euler equation for the risky asset, its rate of return will satisfy,

$$E_t(1 + x_t) = (1 + \rho) \frac{u'(C_t)}{E_t[u'(C_{t+1})]} - \frac{\text{Cov}_t(1 + x_t, u'(C_{t+1}))}{E_t(u'(C_{t+1}))}$$
The Expected Return Premium on the Risky Asset

Taking the difference of the first order conditions for the risky and the risk free asset, we get,

\[ E_t(x_t) - r_t = -\frac{\text{Cov}_t(1 + x_t, u'(C_{t+1}))}{E_t(u'(C_{t+1}))} \]

The expected return premium of the risky asset depends on the covariance of the rate of return of the risky asset with the marginal utility of consumption. Given that the marginal utility of consumption is negatively correlated with consumption, because of decreasing marginal utility, the expected return premium of the risky asset will depend positively on the covariance of the rate or return of the risky asset with consumption. Risky assets whose returns are positively correlated with consumption, will tend to have a higher expected return relative to the risk free asset.

This model of the determination of expected asset returns is known as the consumption capital asset pricing model, or, consumption CAPM.
From the First Order Conditions to the Full Analysis of Consumption

• From the first order conditions we cannot fully describe the behavior of consumption and savings, apart from specific cases.

• There are two special cases where we can come up with specific solutions. The first is the case of *insurable income risk*, and the latter is the case of *quadratic utility functions*.

• As demonstrated by Merton (1971), if labor income can be insured, we can deduce specific solutions for consumption for a broad class of utility functions, the so called *hyperbolic absolute risk aversion*, or HARA, utility functions. This class includes isoelastic utility function with constant relative risk aversion (*constant relative risk aversion*, or CRRA), the exponential utility function with constant absolute risk aversion (*constant absolute risk aversion* or CARA) and *quadratic utility functions*.
The Principle of Optimality and the Derivation of Specific Solutions

One way to derive the specific solution is to use the Belmann principle of optimality, which says that for any value of the state variable (the portfolio of assets in this case) at a given time period, the solution for the future must be optimal.

Using this principle and the value function, the solution can be found through backward induction. For example, in period $T-2$, for any value of the portfolio of assets $A_{T-2}$, the household faces a two-period problem. Solving this problem, take a step back, and solve the problem of the period $T-3$, having already identified the value of the value function of $T-2$. Then we move on and solve the same problem inductively for the $T-4$ period and so on.

In the case of an infinite time horizon we take the limit of the solution to the problem of $T$ periods, as $T$ tends to infinity. Alternatively, we can also solve the problem of infinite periods directly.
The Special Case of HARA Utility Functions

• If the consumer has an infinite time horizon, if the "safe" interest rate is fixed and if the “uncertain” return $x_t$ is distributed according to an independent, uniform, probability distribution, the value function is independent of time and only depends on the state variable $A_t$. Therefore, we can presume its form, derive the consumption function and verify if our presumption was correct.

• The reason that HARA type utility functions allow us to infer analytical solutions, is that the value function belongs to the same family as the utility function, and all that remains is to infer the parameters of the value function.
The Case of Logarithmic Preferences

We consider the case in which,

\[ u(C_t) = \ln C_t \]

Using the result of Merton that the value function has the same functional form as the utility function, for HARA type utility functions, we presume that the value function takes the form,

\[ V(A_t) = a \ln(A_t) + b \]

where \( a \) and \( b \) are constant parameters that must be determined.
The Problem of the Consumer with Logarithmic Preferences

This conjecture allows us to formulate the maximization problem in period $t$ as,

$$\max \ln(C_t) + \frac{1}{1 + \rho} E_t \left[ a \ln(A_{t+1}) + b \right]$$

under the constraint,

$$A_{t+1} = (A_t - C_t) \left[ (1 + r) \omega_t + (1 + x_t)(1 - \omega_t) \right]$$

We have also used the extra assumption that $Y=0$. The first order conditions imply that,

$$C_t = \left( 1 + \frac{a}{1 + \rho} \right)^{-1} A_t \quad E_t \left[ (r - x_t) \left( (1 + r) \omega + (1 + x_t)(1 - \omega) \right)^{-1} \right] = 0$$
Interpretation of the First Order Conditions and Derivation of the Consumption Function

The first condition determines consumption as a linear function of the value of the total portfolio of assets of the household. The second determines indirectly the optimal proportion invested in the “safe” asset $\omega$ as a constant, due to the assumption that the return of the “risky” asset $x$ is distributed according to a uniform, independent probability distribution. The proportion $\omega$ is independent of the total value of the portfolio of assets.

In order to find $a$ and $b$ we substitute in the value function and compare coefficients with. $b$ is a complex but not economically interesting constant, which depends on all the parameters of the model. $a$ is determined as $(1+\rho)/\rho$. Substituting this value in the consumption function, we get,

$$a = \frac{1+\rho}{\rho} \text{ therefore, } C_t = \frac{\rho}{1+\rho} A_t$$

Consumption is a linear function of the total value of the portfolio of the household, and the marginal propensity to consume out of wealth depends only on the pure rate of time preference and not on the real interest rate or the rate of return of the “risky” asset.
The Case of Quadratic Preferences and Certainty Equivalence

The second case we shall examine is the case of quadratic preferences. We shall assume that the portfolio consists only of the “safe” asset. The maximization problem of the household is,

$$\max E_0 \left[ \sum_{t=0}^{T-1} \left( \frac{1}{1+\rho} \right)^t (aC_t - bC_t^2) \right]$$

under the constraint,

$$A_{t+1} = (1+r)(A_t + Y_t - C_t)$$
From the First Order Conditions to the Full Analysis of Consumption

From the first order conditions,

\[ E_tC_{t+1} = \frac{r - \rho}{1 + r} \frac{a}{2b} + \frac{1 + r}{1 + \rho} C_t \]

In what follows we shall assume that \( r = \rho \). In this case the first order conditions take the form,

\[ E_tC_{t+1} = C_t \]

Therefore, it follows that,

\[ E_0C_t = C_0 \]

for \( t=0,1,2,...,T-1 \). Expected consumption is constant for the duration of the program (consumption smoothing).
The Inter-temporal Budget Constraint and the Determination of Consumption

The inter-temporal budget constraint is given by,

$$\sum_{t=0}^{T-1} \left( \frac{1}{1+r} \right)^t C_t = A_0 + \sum_{t=0}^{T-1} \left( \frac{1}{1+r} \right)^t Y_t$$

As a result, at time 0 the household must satisfy,

$$E_0 \left[ \sum_{t=0}^{T-1} \left( \frac{1}{1+r} \right)^t C_t \right] = A_0 + E_0 \left[ \sum_{t=0}^{T-1} \left( \frac{1}{1+r} \right)^t Y_t \right]$$

Substituting for consumption and taking the limit as $T$ tends to infinity,

$$C_0 = \frac{r}{1+r} \left( A_0 + E_0 \left[ \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t Y_t \right] \right)$$
The Permanent Income Hypothesis
Properties of the Consumption Function with Quadratic Preferences

• Consumption is constant and is a fixed percentage of the total wealth of the household, including the present value of expected labor income.

• In every period, the household consumes a constant fraction of its total wealth, depending on the real interest rate (or the pure rate of time preference), so that expected total wealth remains constant.

• The change in consumption from period to period is determined only by the revision of expectations regarding labor income.
Changes in Labor Income and Changes in Consumption

Only previously unanticipated changes in labor income bring about changes in consumption,

\[
C_t - C_{t-1} = \frac{r}{1 + r} \sum_{i=0}^{\infty} \left( \frac{1}{1 + r} \right)^i \left( E_t(Y_{t+i}) - E_{t-1}(Y_{t+i}) \right)
\]

Under the hypothesis that income follows an AR(1) process of the form,

\[
Y_t = Y_0 + \lambda Y_{t-1} + \varepsilon_t
\]

the change in consumption is determined by,

\[
C_t - C_{t-1} = \frac{r}{1 + r - \lambda} \varepsilon_t
\]
Temporary versus Permanent Changes in Labor Income and Consumption

• If $\lambda < 1$, disturbances in labor income are temporary, and the coefficient of the unanticipated change in labor income $\varepsilon$ is smaller than unity. This case incorporates the predictions of the “permanent income” hypothesis of Friedman (1957) and the “life cycle” hypothesis of Modigliani και Brumberg (1954), that consumption smooths out transitory changes in income.

• If $\lambda = 1$, then disturbances in labor income $\varepsilon$ are of a permanent nature, and the coefficient of the change in consumption is also equal to unity. Permanent changes in labor income lead to equivalent permanent changes in household consumption.

• Empirical studies of the “permanent income” hypothesis suggest that aggregate consumption displays “excess sensitivity” to changes in current income, indicating that one may have to go beyond the “permanent income” hypothesis in explaining aggregate consumption.
The Consumption Capital Asset Pricing Model with Quadratic Preferences

Assuming quadratic preferences, the marginal utility of consumption is given by,

\[ u'(C_t) = a - bC_t \]

The marginal utility of consumption is thus a negative linear function of consumption. Substituting for the marginal utility of consumption in consumption CAPM, we get,

\[ E_t(x_t) - r_t = \frac{2b \text{Cov}_t (1 + x_t, C_{t+1})}{a - bE_tC_{t+1}} \]

Under quadratic preferences the expected return premium of a risky asset is proportional to the covariance of its return with consumption. This factor of proportionality is sometimes referred to as a consumption beta, from a regression of consumption growth on asset returns.
The Consumption Capital Asset Pricing Model and the Equity Premium Puzzle

• The original capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) assumed that investors are concerned with the mean and variance of the return of their portfolio, rather than the mean and variance of consumption. That version of the model thus focused on so-called market betas, that is coefficients from regressions of asset returns on the returns of a market portfolio.

• A central prediction of the consumption CAPM is that the return premium of a risky asset is proportional to its consumption beta.

• Following Mehra and Prescott (1985), empirical studies suggest a so called “equity premium puzzle”, i.e a much bigger difference between the average return of equities (the risky asset) and government bonds (the safe asset) than would be suggested by the consumption CAPM.
Some Conclusions about Consumption under Uncertainty

• We have examined the determination of household consumption under conditions of uncertainty, in conjunction with the determination of the allocation of the portfolio of the household among alternative assets (Samuelson 1969, Merton 1969).

• Under conditions of uncertainty, for a household that can borrow and lend freely in the capital market, consumption generally depends on the same factors as under certainty. The current and expected future rates of assets, the current and expected future labor income and the total value of the portfolio and human wealth of the household.

• Consumption does not depend on current household income but on total wealth, which consists of the value of its portfolio, plus the present value of current and expected future labor income In this sense, consumption smooths out temporary changes in income, as it depends on “permanent” or “life cycle” income. (Friedman 1957, Modigliani Brumberg 1954).

• However, the “permanent” income hypothesis cannot explain many of the features of individual or aggregate consumption patterns, especially the “excess sensitivity” of consumption to changes in current income. In addition, the consumption capital asset pricing model, which is a associated prediction of stochastic inter-temporal models of consumption, seems to be refuted by the “equity premium” puzzle. The literature has thus also examined more general models that result in precautionary savings, or in which markets are incomplete and households are also bound by borrowing constraints (see Attanasio 1999).