Chapter 11
New Classical Models of Aggregate Fluctuations

In previous chapters we studied the long run evolution of output and consumption, real interest rates and real wages, and the long run evolution of the price level and inflation. In order to focus on long-term trends we made the assumption that all markets are competitive and in continuous equilibrium, through the full adjustment of prices, wages and interest rates.

However, as we noted in Chapter 1, economies are characterized by fluctuations in relation to their long-term trends. In some periods output, consumption and employment grow at high rates, while at other times they grow at low or even negative rates. In some periods unemployment is low and in others quite high. Inflation displays significant fluctuations as well.

Understanding the determinants of aggregate fluctuations is the second main objective of macroeconomics. In this, and the chapters that follow, we present the main theories regarding the nature of aggregate fluctuations.

In this chapter we start by introducing classical models of aggregate fluctuations. “New” classical models are essentially dynamic stochastic general equilibrium models (DSGE), based on optimizing households and firms, flexible wages and prices, and fully competitive markets. Fluctuations in these models are caused by real shocks to productivity, household preferences and government expenditure, and the effects of these shocks are propagated through endogenous dynamic processes, such as consumption and investment.

We start with the so called stochastic growth model, which is an extended stochastic version of the Ramsey model. The utility function of a representative household depends on both consumption of goods and services, and leisure, while random disturbances to real factors, such as productivity, preferences and government expenditure cause aggregate fluctuations.

To be able to analyze the model, we make simplifying assumptions regarding the production and utility functions. Without them the model becomes extremely complicated. The dynamic analysis is conducted in discrete rather than continuous time.

After analyzing and characterizing this generalized “new classical” model, we also analyze a short term version of the model, without capital accumulation. In this model, labor is the only variable factor of production. This model, is comparable to the “new keynesian” models we shall analyze in subsequent chapters, and will help us pinpoint the main differences between the “new classical” and the “new keynesian” approaches.

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1 This approach owes a lot to the papers by Kydland and Prescott (1982), Long and Plosser (1983) and Prescott (1986). It is surveyed in King and Rebelo (1999). As we shall see, this approach has also influenced the “new keynesian” approach to aggregate fluctuations, which now also relies on dynamic stochastic general equilibrium models (DSGE).
As we shall see, in almost all classical models, the main impulses that generate aggregate fluctuations are real, i.e., shocks to productivity, preferences and government expenditure. This is why this class of models is often referred to as the real business cycle model (RBC). Monetary shocks have no real effects on output, employment and capital accumulation in this class of models, and only affect real money balances, and nominal variables such as the price level, inflation and nominal wages and interest rates.

11.1 The Nature and Key Characteristics of Aggregate Fluctuations

As we have seen in Chapter 1, aggregate fluctuations are not characterized by some simple repetitive regularity and seem to be characterized by randomness. The prevailing view today, which dates back to Frisch (1930) and Slutsky (1937), is that economies are subject to various kinds of random disturbances, which, through the operation of economic transmission mechanisms, affect output, employment, real wages, real interest rates, the price level and inflation, and set in motion dynamic stochastic adjustment processes.

The way in which modern macroeconomists approach and analyze economic fluctuations owes a lot to the following important observation of Lucas (1977):

Technically, movements about trend in gross national product in any country can be well described by a stochastically disturbed difference equation of very low order. These movements do not exhibit uniformity of either period or amplitude, which is to say, they do not resemble the deterministic wave motions which sometimes arise in the natural sciences. Those regularities which are observed are in the co-movements among different aggregative time series … One is led by the facts to conclude that, with respect to the qualitative behavior of co-movements among series, business cycles are all alike. To theoretically inclined economists, this conclusion should be attractive and challenging, for it suggests the possibility of a unified explanation of business cycles, grounded in the general laws governing market economies, rather than in political or institutional characteristics specific to particular countries or periods. (p. 9-10).

This observation by Lucas caused significant changes in the way in which all macroeconomic schools of thought are now approaching and try to explain aggregate fluctuations. The traditional macroeconomic and macro-econometric models, from the 1950s to the 1970s, were deemed to have a number of weaknesses in the detailed study of economic fluctuations, and the impact of monetary and fiscal policy in relation to the criteria of Lucas.

The most important of the weaknesses of traditional macroeconomic and macro-econometric models was that the macroeconomic relationships assumed were not explicitly drawn from well defined microeconomic foundations, based on inter-temporal optimization on the part of households and firms. Therefore one could not easily interpret their parameters, and be sure of their stability. This was the basis of the famous, and very potent, Lucas critique of econometric policy evaluation.²

² See Lucas (1976) who concluded that “given that the structure of all econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models.” (p. 41). He came to this conclusion after having demonstrated the fragility of traditional econometric models of aggregate consumption, investment and the Phillips curve.
The realization of these weaknesses, which applied equally to keynesian and monetarist models, gradually gave rise to alternative models based on more satisfactory dynamic microeconomic foundations.

11.2 The Stochastic Growth Model

The first model we shall focus on is the so-called stochastic growth model. This is a competitive dynamic stochastic general equilibrium model, without externalities, asymmetric information, frictions and other imperfections of markets.

This model is essentially a generalization of the Ramsey model. It not only excludes any market imperfections, but also all issues related to the heterogeneity of economic agents. The extended Ramsey model is therefore the natural starting point for the study of aggregate fluctuations, like the original Ramsey model is the “natural” starting point for the study of the long run growth.

There are two directions in which the Ramsey must be extended in order to study aggregate fluctuations.

First, one should allow for random disturbances, which can cause fluctuations. Without random disturbances, the Ramsey model converges to a unique steady state. The disturbances usually introduced in the Ramsey model are disturbances in total factor productivity (technology shocks), as well as real demand shocks, such as shocks to the preferences of consumers or real government expenditure. Since both kinds of shocks are real - unlike monetary or nominal shocks - this model turns out to be a real business cycle model (RBC).

Second, in order to allow the Ramsey model to explain fluctuations not only in total output, but also employment, employment must become endogenous. This is achieved through the introduction of employment in the utility function of a representative household, in order to establish an endogenous labor supply function.

The extended Ramsey model which we end up with is a dynamic stochastic general equilibrium model (DSGE), in which fluctuations are caused by real shocks.

There are a number of identical households and firms, so this is a competitive representative household model. Firms use labor and capital in order produce a homogeneous product. They choose investment and employment in order to maximize their profits, while households choose consumption and labor supply in order to maximize their inter-temporal utility.

The key variables and parameters of the model are as follows:

- $Y$ total output
- $K$ physical capital
- $L$ employment
- $A$ labor efficiency (productivity)
- $C$ total private consumption
- $C^G$ total real government expenditure
- $N$ total population
- $\delta$ depreciation rate of capital
11.2.1 The Behavior of Firms

The economy consists of a large number of identical households and firms, interacting through competitive markets. Output and factor prices are thus given for every household and every firm.

The representative firm has a production function with constant returns to scale, which takes the Cobb-Douglas form. Thus, the aggregate production function is also Cobb Douglas.

\[ Y_t = K^\alpha (A_t L_t)^{1-\alpha} \quad 0<\alpha<1 \]  

(11.1)

The demand for output consists of private consumption, investment and government consumption. Government consumption is financed through non distortionary taxation, and, in each period, taxes are equal to government consumption. Thus, the equilibrium condition in the output market is given by,

\[ Y_t = C_t + C_t^G + K_{t+1} - K_t + \delta K_t \]  

(11.2)

Solving (11.2) for \( K_{t+1} \), we get a capital accumulation equation of the form,

\[ K_{t+1} = K_t + Y_t - C_t - C_t^G - \delta K_t \]  

(11.3)

To the extent that total savings \( Y-C-C^G \) exceed depreciation investment \( \delta K \), there is accumulation of capital.

Labor and capital are paid their marginal product, as firms maximize profits taking the real interest rate \( r \) and the real wage \( W \) as given.

\[ W_t = (1-\alpha) \left( \frac{K_t}{A_t L_t} \right)^\alpha A_t \]  

(11.4)

\[ r_t = \alpha \left( \frac{A_t L_t}{K_t} \right)^{1-\alpha} - \delta \]  

(11.5)

(11.1)-(11.5) describe the behavior of firms. Firms employ workers up to the point where the marginal product of labor equates the real wage, and capital up to the point where the marginal product of capital equals the user cost of capital, equal to the real interest rate plus the depreciation rate.

11.2.2 The Representative Household
The economy is inhabited by a large number of identical households, each of which has an infinite time horizon. The representative household maximizes its expected inter-temporal utility function, which depends on the path of real consumption of goods and services and leisure. The utility function is defined by,

\[ U = E_t \sum_{s=0}^{\infty} \left( \frac{1}{1+\rho} \right)^s u(c_{t+s}, 1-l_{t+s}) \frac{N_{t+s}}{H} \]  \hspace{1cm} (11.6)

where \( E \) is the mathematical expectations operator and \( u \) is the per capita instantaneous utility of the representative household. Per capita consumption is given by \( c = \frac{C}{N} \) and per capita employment by \( l = \frac{L}{N} \). We shall assume that the instantaneous utility function is log-linear.

\[ u_t = \ln c_t + b \ln(1-l_t), \hspace{1cm} b > 0 \]  \hspace{1cm} (11.7)

This assumption is made in order to arrive at simpler functional relationships. However, like all simplifications, this assumption implies specific properties for the model.

11.2.3 Population, Efficiency of Labor and Government Expenditure

Population increases exogenously at a rate \( n \) per period. Consequently,

\[ \ln N_t = \tilde{n} + nt, \hspace{1cm} n < \rho \]  \hspace{1cm} (11.8)

The final assumptions of the model concern the behavior of the two main exogenous variables. Both productivity (labor efficiency), and government expenditure are supposed to be subject to random disturbances.

The stochastic process describing the evolution of the efficiency of labor is given by\(^3\)

\[ \ln A_t = a + gt + a_t \]  \hspace{1cm} (11.9)

where,

\[ a_t = \eta_A a_{t-1} + \epsilon^A_t, \hspace{1cm} -1 < \eta_A < 1 \]  \hspace{1cm} (11.10)

\( \epsilon^A \) follows a white noise stochastic process.

(11.9) and (11.10) imply that labor efficiency grows at an exogenous rate \( g \), but that it is subject to random disturbances \( a \) that follow a stationary first order autoregressive process. The assumptions about (11.10) imply that the impact of a technological disturbance is gradually reduced over time.

Similar assumptions are made regarding the stochastic process that describes the evolution of real government expenditure. We assume that real government expenditure is growing at an average rate

\(^3\) Mathematical Annex 4 contains an introduction to stochastic processes.
$n+g$, i.e. that on average it remains constant relative to total output. However, we also assume that real government expenditure is subject to disturbances follow a stationary first order autoregressive process. More particularly,

$$\ln G_t = c^s + (n + g)t + \epsilon_t^G$$  \hspace{1cm} (11.11)

where,

$$c_t^G = \eta c_{t-1}^G + \epsilon_t^G \hspace{1cm} -1 < \eta < 1$$  \hspace{1cm} (11.12)

$\epsilon^G$ follows a white noise stochastic process.\(^4\)

These elements complete the structure of the model. The two most important differences from the original Ramsey model are, first, the introduction of leisure time in the utility function of the representative household, which potentially allows for fluctuations in employment, and, second, the introduction of random disturbances to labor efficiency (productivity) and government expenditure, which lead to fluctuations around a long-term trend.

Before we look at the general properties of the model, it is worth considering the implications for the behavior of the representative household of the introduction of leisure in the utility function, as well as the implications of uncertainty, in the form of random disturbances.

11.2.4 Labor Supply of the Representative Household

The first difference of this model from the Ramsey model arises from the introduction of leisure time in the utility function of the household, which makes labor supply endogenous. To analyze the importance of this addition, let us first consider the problem of a household that lives for a single time period and has no assets. The problem of that household is defined as the maximization of,

$$\ln c + b \ln(1 - l)$$

under the constraint $c = Wl$.

The Lagrangian is defined by,

$$\Lambda = \ln c + b \ln(1 - l) + \lambda (Wl - c)$$  \hspace{1cm} (11.13)

The first order conditions for $c$ and $l$ are,

$$\frac{1}{c} - \lambda = 0$$  \hspace{1cm} (11.14)

$$-\frac{b}{1 - l} + \lambda W = 0$$  \hspace{1cm} (11.15)

\(^4\) The assumption that the processes driving labor productivity and real government expenditure are AR(1) are made for simplicity, and can of course be generalized.
From the budget constraint \( c = Wl \) and (11.14) it follows that \( \lambda = l/Wl \). Substituting in (11.15), we get that,

\[
- \frac{b}{1-l} + \frac{1}{l} = 0
\]

(11.16)

From (11.16) it is apparent that labor supply is independent of the real wage. This is because of the assumption of logarithmic preferences, implying that the elasticity of substitution between consumption and leisure is equal to unity. Thus, the substitution effect from a change in the real wage is counteracted by the income effect. However, this does not mean that temporary changes in real wages do not affect labor supply. This can be seen if we look at the behavior of a household living for two periods.

11.2.5 Inter-temporal Substitution in Labor Supply

We shall next analyze the behavior of a household living for two periods, has no initial wealth, and no uncertainty about the real interest rate or the real wage of the second period.

The inter-temporal budget constraint of the household is given by,

\[
c_1 + \frac{1}{1+r} c_2 = W_1 l_1 + \frac{1}{1+r} W_2 l_2
\]

(11.17)

The Lagrangian is defined by,

\[
\Lambda = \ln c_1 + b \ln (1-l_1) + \frac{1}{1+\rho} (\ln c_2 + b \ln (1-l_2)) + \lambda \left[ W_1 l_1 + \frac{1}{1+r} W_2 l_2 - c_1 - \frac{1}{1+r} c_2 \right]
\]

The household chooses consumption and labor supply for each of the two periods. From the first order conditions for labor supply,

\[
\frac{b}{1-l_1} = \lambda W_1
\]

(11.18)

\[
\frac{b}{1-l_2} = \frac{1+\rho}{1+r} \lambda W_2
\]

(11.19)

Dividing (11.19) by (11.18),

\[
\frac{1-l_1}{1-l_2} = \frac{1+\rho}{1+r} \frac{W_2}{W_1}
\]

(11.20)

(11.20) implies that the relative labor supply in the two periods depends positively on the relative real wage in the two periods. The higher the real wage of the first period in relation to the real wage of the second period, the higher the labor supply of the first period, in relation to that of the second. The household substitutes labor between periods, depending on relative real wages between
periods. Because of logarithmic preferences, the inter-temporal substitution elasticity is equal to one.

Moreover, the higher the real interest rate $r$ the greater the labor supply of the first period compared to the second period. The increase in the interest rate increases the attractiveness to work in the first period and save for the second period, compared to working in the second period. It has the opposite effect of the pure rate of time preference rate $\rho$.

These effects of relative wages over time and the real interest rate on labor supply are known as *inter-temporal substitution in labor supply*. Such effects obviously generalize to a multi period setting. Consequently, fluctuations in real wages and the real interest rate can cause fluctuations in employment, although permanent changes in real wages do not affect labor supply in a model with logarithmic preferences.\(^5\)

### 11.2.6 Uncertainty and the Behavior of the Representative Household

The second element that differentiates the stochastic growth model from the Ramsey model is uncertainty arising from the stochastic disturbances. Therefore, the expectations of the representative household for future developments play a significant role.

It can be shown that, for the general case when the household maximizes the expected inter-temporal utility function (11.6), the Euler equation for consumption takes the form,

$$
\frac{1}{c_t} = \frac{1}{1 + \rho} \frac{E_t^f}{c_{t+1}} \left[ \frac{1}{c_{t+1}} (1 + r_{t+1}) \right] \tag{11.21}
$$

The mathematical expectation of the product of two random variables is not equal to the product of mathematical expectations. It is equal to the product of mathematical expectations plus the covariance of two random variables. Thus, (11.21) implies,

$$
\frac{1}{c_t} = \frac{1}{1 + \rho} \left\{ E_t \left[ \frac{1}{c_{t+1}} \right] E_t (1 + r_{t+1}) + \text{Cov} \left( \frac{1}{c_{t+1}}, (1 + r_{t+1}) \right) \right\} \tag{11.22}
$$

On the other hand, from the first-order conditions for consumption and labor supply, the ratio of consumption to leisure is a positive function of the real wage of the form,

$$
\frac{c_t}{1 - l_t} = \frac{W_t}{b} \tag{11.23}
$$

(11.23) links labor supply (leisure) and consumption with the real wage. It includes only current variables, as there is no uncertainty in the current period.

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\(^5\) The concept of inter-temporal substitution in labor supply was first analyzed in an important paper by Lucas and Rapping (1969). For an empirical investigation of its significance for fluctuations in employment in the USA and the UK see Alogoskoufis (1987a,b).
Equations (11.21) and (11.23) are the basic equations that describe the behavior of households in this model.

We can now examine the properties of the model. This model is not easy to solve analytically, as it contains factors that are linear, and factors that are log-linear in its variables. The properties of the model can be described if we simplify it further, or if we use a log-linear approximation around the balanced growth path, and solve it numerically for specific values of the parameters.

In the Annex to this chapter, we present the Campbell (1994) log-linear approximation of the full model, around its balanced growth path. This allows us to describe the full properties of the model through a numerical simulation.

In the remainder of this section we shall concentrate on the properties of a simplified version of the model.

11.2.7 A Simplified Version of the Stochastic Growth Model

To further analyze the stochastic growth model, we will consider a special case without government expenditure and a depreciation rate of 100%. The equations that describe the accumulation of capital and the determination of the real interest rate are then simplified to,

\[
k_{t+1} = Y_t - C_t \tag{11.24}\]

\[
1 + r_t = \alpha \left( \frac{A L_t}{K_t} \right)^{1-\alpha} \tag{11.25}\]

Because of the assumption of competitive markets and the absence of externalities, the equilibrium of the model is Pareto optimal. We shall define the properties of the model by solving for the competitive equilibrium.

We will focus on two variables. Labor supply per person \(l\), and the savings rate \(s\). Defining the savings rate we also determine aggregate consumption as, \(C = (1-s)Y\).

We will focus on both behavioral equations of the representative household (11.21) and (11.23). Once we determine labor supply and the savings rate, all the rest follows automatically either from equilibrium conditions or definitions.

From (11.21), after we use the relations,

\[
c_t = (1-s_t)Y_t / N_t, \quad 1 + r_{t+1} = \alpha Y_{t+1} / K_{t+1}, \quad K_{t+1} = s_t Y_t
\]

we end up that the savings rate is constant and given by,

\[
\frac{s}{s} = \frac{\alpha (1+n)}{1+\rho} \tag{11.26}\]
Because of logarithmic preferences, the savings rate is independent of the real interest rate and constant.

From (11.23), after we note that,

\[
c_t = (1 - \bar{s}) Y_t / N_t, \quad W_t = (1 - \alpha) Y_t / (l_t N_t)
\]

we end up with the conclusion that labor supply per household member is constant and given by,

\[
\ln Y_t = \alpha \ln K_t + (1 - \alpha)(\ln A_t + \ln L_t)
\]  

(11.28)

Labor supply is constant because the impact of the shocks in technology (labor efficiency) on the real wage and the real interest rate cancel each other out, so there is no inter-temporal substitution. This is due to the specific assumption that we made in order to simplify the model, and as one can see from the analysis of the full model in the Annex is not a general feature of the model.

We can now determine fluctuations in total output. Log-linearizing the production function (11.1) we get,

\[
\ln Y_t = \alpha \ln s + \alpha \ln Y_{t-1} + (1 - a)(\ln A_t + \ln l + \ln N_t)
\]  

(11.29)

We can substitute the logarithm of \( A \) and \( N \) from equations (11.8) and (11.9). This implies,

\[
\ln Y_t = \alpha \ln s + \alpha \ln Y_{t-1} + (1 - \alpha) \left[ (\alpha + gt) + v_t + (\ln l + n + nt) \right]
\]  

(11.30)

We can express (11.30), as,

\[
y_t = \alpha y_{t-1} + (1 - \alpha) a_t
\]  

(11.31)

where,

\[
y_t = \ln Y_t - \ln Y_t, \text{ are percentage deviations of output from trend output.}
\]

The logarithm of trend output, is defined as,

\[
\ln \bar{Y}_t = \frac{\alpha}{1 - \alpha} \ln s + \ln l + \bar{a} + \bar{n} + (n + g)t
\]

From (11.10) and (11.31), we end up with,
From (11.32), the percentage deviations of total real output from trend follow a second order autoregressive process (AR(2)). Because \( \alpha \) is low (about \( \frac{1}{3} \)), the dynamic behavior of total real output depends primarily on the degree of persistence of productivity shocks. If the persistence of productivity shocks \( \eta_A \) is high, then we have considerable persistence in the fluctuations of output. Otherwise, the persistence of output fluctuations around trend is low.

For example, running a regression of the log of US real GDP on a linear trend, and its two lags, using annual data for the period 1890-2014, one gets,

\[
\log(GDP)_t = 0.270 + 1.211 \log(GDP)_{t-1} - 0.319 \log(GDP)_{t-2} + 0.0036 t
\]

\[\begin{array}{ccc}
(0.083) & (0.086) & (0.087) & (0.0012)
\end{array}\]

\( R^2 = 0.999 \), \( \text{DW} = 2.020 \), \( T = 125 \)

From this regression, which seems to fit the data for the US real GDP quite well, the estimate of \( \eta_A \) is equal to 0.824 (s.e. 0.082), and the estimate of \( \alpha \) is equal to 0.386 (s.e. 0.132). The estimate of \( g + \eta \), the long run growth rate, is equal to 0.033 (s.e. 0.001). From these estimates, this simplified stochastic growth model can account for fluctuations in US GDP if the persistence of productivity shocks is of the order of 0.8.

If the degree of persistence of productivity shocks is zero, then (11.32) would simplify to,

\[ y_t = \alpha y_{t-1} + (1-\alpha)\epsilon_t^A \]  \hspace{1cm} (11.33)

which obviously cannot account for the fluctuations of US real GDP.

The simplified form of the model, summarized in equation (11.32), contains many of its essential elements, and provides the basic “new classical” account of fluctuations in total output (GDP) around trend, mainly on the basis of persistent productivity shocks and capital accumulation.

However, many other features of aggregate fluctuations are not adequately described by this simplified version of the stochastic growth model.

1. **The constant savings ratio.** This means that consumption will display the same degree of variability as output and investment, which does not tend to happen in reality.
2. **The constant employment rate.** In reality, the employment rate is not constant over the business cycle. Employment is pro-cyclical, moving in the same direction as output.
3. **Real Wages over the Business Cycle.** In the simplified stochastic growth model real wages are pro-cyclical and equally volatile as GDP per capita, which is not always the case.

When one examines the more general form of the model, assuming a low depreciation rate, as we do in the Annex to this chapter, many of these weaknesses are corrected, as savings, investment and

\[^{\text{6}}\text{Standard errors are in parentheses below estimated coefficients. } R^2 \text{ is the coefficient of determination, } \text{DW is the Durbin Watson statistic and } T \text{ the number of observations.}\]
employment also display fluctuations in response to productivity shocks. For example, in the full stochastic growth model, analyzed in the Annex, the savings rate is not constant, and consumption tends to be less variable than investment and output. In addition, in the full stochastic growth model, the employment rate is pro-cyclical, and moves in the same way as output. Moreover, the introduction of public expenditure shocks or preference shocks could relax the strict dependence of fluctuations in real wages on fluctuations in aggregate productivity.

11.3 A “New Classical” Model without Capital

In what follows we shall focus on an analytically simpler version of the “new classical” model of aggregate fluctuations, in which the only variable factor of production is labor. We shall thus abstract from capital accumulation.

In this analytically simpler model we allow for a more general approach to the preferences of the representative household, and also distinguish between nominal and real variables, in order to consider the determination of the level of prices and wages, inflation and nominal interest rates, and the role of monetary factors in classical models.7

11.3.1 The Representative Household

The representative household is assumed to maximize,

$$E_t \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t u(C_{t+1}, L_{t+1})$$

where $C$ is consumption and $L$ is labor supply. We assume that,

$$u_{C_t} = \frac{\partial u}{\partial C_t} > 0, \quad u_{C_tC_t} = \frac{\partial^2 u}{\partial C_t^2} \leq 0, \quad u_{L_t} = \frac{\partial u}{\partial L_t} \leq 0, \quad u_{L_tC_t} = \frac{\partial^2 u}{\partial C_t \partial L_t} \leq 0. \quad (11.35)$$

The constraints under which the maximization takes place are given by,

$$P_t C_t + \frac{1}{1+i_t} B_t \leq B_{t-1} + W_t L_t - T_t \quad (11.36)$$

$$\lim_{T \to \infty} E_t B_T \geq 0 \quad (11.37)$$

where $P$ is the price level, $W$ the nominal wage, $i$ the nominal interest rate, $B$ a nominal one period bond, and $T$ an exogenous transfer of nominal income to the household (dividends, government transfers of taxes).

From the first order conditions it follows that,

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7 This is essentially a “short run” classical model, the “short run” being defined as the time span for which the capital stock is fixed. In the “short run” firms care about the utilization rate of their capital stock, varying employment, which is the only variable factor of production. For a more extensive treatment of this short run “new classical” model, see Gali (2008). This simplified model will in future chapters allow us to directly compare the “new classical” with the “new keynesian” approach.
We assume that the per period utility function is given by,

\[ U(C_t, L_t) = \frac{C_t^{1-\theta}}{1-\theta} - \frac{L_t^{1+\lambda}}{1+\lambda} \]

where \( \theta > 0 \) and \( \lambda > 0 \) (11.40)

The first order conditions for the problem of the representative household in this case take the form,

\[ \frac{W_t}{P_t} = C_t^\theta L_t^\lambda \] (11.41)

\[ \frac{1}{1+i} = \frac{1}{1+\rho} E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\theta} \frac{P_t}{P_{t+1}} \right\} \] (11.42)

(11.41) and (11.42) can be written in log-linear form, as,

\[ w_t - p_t = \theta c_t + \lambda l_t \] (11.43)

\[ c_t = E_t(c_{t+1}) - \frac{1}{\theta} \left( i_t - E_t(\pi_{t+1}) - \rho \right) \] (11.44)

where \( w = \ln W, p = \ln P, c = \ln C, l = \ln L \) and \( \pi_t = p_t - p_{t-1} \) is the inflation rate.

11.3.2 The Representative Firm

Production of the representative firm is a positive function of employment, and is described by an aggregate production function of the form,

\[ Y_t = A_t L_t^{1-\alpha} \] (11.45)

where \( A > 0 \) and \( 0 < \alpha < 1 \) are exogenous technological parameters. \( \alpha \) is a constant, while \( A \) follows an exogenous stochastic process.

The representative firm chooses employment in order to maximize profits, for given nominal wages and prices. Profits are determined by,

\[ P_t Y_t - W_t L_t \] (11.46)
Profit maximization implies that employment will be determined so as to equate the marginal product of labor to the real wage. Thus,

\[
\frac{W_t}{P_t} = (1 - \alpha)A_tL_t^{\alpha} \tag{11.47}
\]

One can solve the marginal productivity condition for the price level. The interpretation is that the product price is equal to marginal cost.

\[
P_t = \frac{W_t}{(1 - \alpha)A_tL_t^{\alpha}} \tag{11.49}
\]

Log-linearizing the first order condition (11.47) we get,

\[
w_t - p_t = a_t - \alpha l_t + \ln(1 - \alpha) \tag{11.50}
\]

where \(a = \ln A\).

Log-linearizing the production function (11.45) we get,

\[
y_t = a_t + (1 - \alpha)l_t \tag{11.51}
\]

Having determined the behavior of households and firms, we can now analyze the equilibrium in this model.

### 11.3.3 General Equilibrium

In the basic form of this model we shall assume that there is no investment or public consumption. Accordingly, in product market equilibrium, consumption will be equal to total output.

\[
y_t = c_t \tag{11.52}
\]

The equilibrium condition (11.52) will determine the real interest rate.

The condition for equilibrium in the labor market will require that labor demand, as implied by (11.50), should be equal to labor supply, as implied by (11.43). This will determine the real wage.

The model consists of equations (11.43), (11.44), (11.50) and (11.51) and the equilibrium condition (11.52), and determines employment, output, consumption, real wages and the real interest rate, as functions the exogenous labor productivity \(a\).

The real interest rate is defined by the Fisher equation, as,\(^8\)

\(^8\) To quote from Fisher (1896), “When prices are rising or falling, money is depreciating or appreciating relative to commodities. Our theory would therefore require high or low interest according as prices are rising or falling, provided we assume that the rate of interest in the commodity standard should not vary.” (p. 58). The rate of interest in the commodity standard is the real interest rate, and rising or falling prices are expected inflation. The Fisher equation was further elaborated in Fisher (1930), where it was made even clearer that Fisher referred to expected inflation.
Solving the model for the five endogenous variables, we get,
\[ l = \eta_L a + \tilde{l} \]  
(11.54)
where, \( \eta_L = \frac{1 - \theta}{\theta(1 - \alpha) + \alpha + \lambda} \) and, \( \tilde{l} = \frac{\ln(1 - \alpha)}{\theta(1 - \alpha) + \alpha + \lambda} \).

\[ y = c = \eta_Y a + \tilde{y} \]  
(11.55)
where, \( \eta_Y = 1 + (1 - \alpha) \eta_L = \frac{1 + \lambda}{\theta(1 - \alpha) + \alpha + \lambda} \) and, \( \tilde{y} = (1 - \alpha) \tilde{l} \).

\[ w - p = \eta_W a + \tilde{\omega} \]  
(11.56)
where, \( \eta_W = 1 - \alpha \eta_L = \frac{\theta + \lambda}{\theta(1 - \alpha) + \alpha + \lambda} \) and, \( \tilde{\omega} = (\theta(1 - \alpha) + \lambda) \tilde{l} \).

\[ r = \rho + \theta \eta_Y E_a(\Delta a_t) \]  
(11.57)

(11.54), (11.55), (11.56) and (11.57), along with the product market equilibrium condition (11.52), determine the five endogenous variables, as a function of the exogenous shock to labor productivity \( a \).

It worth noting that fluctuations in employment, output, consumption and real wages are a function only of fluctuations in labor productivity and fluctuations in the real interest rate depend on fluctuations in the expected rate of change of productivity.

Output, consumption and real wages are positive functions of productivity, while employment is a positive function of productivity only if \( \theta < 1 \), i.e. if the inter-temporal elasticity of substitution of consumption is greater than one. If \( \theta > 1 \), employment is a negative function of productivity, while if \( \theta = 1 \), employment is independent of productivity. This is because if \( \theta < 1 \) the substitution effect dominates over the income effect, after a change in productivity and real wages, and employment rises. If \( \theta > 1 \) the income effect dominates over the substitution effect, while in the case \( \theta = 1 \) the two effects cancel each other out, and employment is not affected.

Only real factors, such as real productivity, affect fluctuations in real variables. As in the stochastic growth model, monetary factors such as money supply and nominal interest rates have no impact on the evolution of real variables.

11.4 Monetary Factors in the “New” Classical Model
In order to examine the impact of monetary factors in the “new” classical model, we shall assume the existence of a money demand function by households and firms, which, in logarithms, takes the form,

\[ m_t - p_t = y_t - \eta i_t \]  \hspace{1cm} (11.58)

where \( \eta \) is the semi-elasticity of money demand with respect to the nominal interest rate.

From the definition of the real interest rate through the Fisher equation (11.53), the nominal interest rate is equal to,

\[ i_t = r_t + E_t(\pi_{t+1}) \]  \hspace{1cm} (11.59)

where the real interest rate \( r_t \) is determined by (11.57) and is independent of monetary factors.

We will show that, like in the case of the models analyzed in Chapter 10, when the central bank follows a rule for the money supply, then the model determines the price level and the level of inflation and nominal interest rates. If the central bank pegs the nominal interest rate, the price level and the level of the nominal money supply cannot be determined, unless the nominal interest rate reacts sufficiently strongly to changes in the price level.

11.4.1 Exogenous Path for the Money Supply

If the central bank determines an exogenous path for the money supply, then, from (11.58) and (11.59) it follows that,

\[ p_t = \frac{\eta}{1+\eta} E_t(p_{t+1}) + \frac{1}{1+\eta} m_t - \frac{1}{1+\eta} (y_t - \eta r_t) \]  \hspace{1cm} (11.60)

Under the assumption that \( \eta > 0 \), the solution of (11.59) implies that,

\[ p_t = \frac{1}{1+\eta} \sum_{j=0}^{\infty} \left( \frac{\eta}{1+\eta} \right)^j E_t \left( m_{t+j} - y_{t+j} + \eta r_{t+j} \right) \]  \hspace{1cm} (11.61)

From (11.61), the price level and inflation are determined as functions of the exogenous path of the money supply, and the paths of real output and the real interest rate, which, as we have seen, are independent of monetary factors in the “new” classical model. The nominal interest rate is determined endogenously from (11.61) and (11.59).

11.4.2 Exogenous Path for the Nominal Interest Rate

If we assume that the central bank follows an exogenous path for the nominal interest rate, then, from the Fisher equation (11.59), it follows that,

\[ E_t(\pi_{t+1}) = i_t - r_t \]  \hspace{1cm} (11.62)
(11.62) does not determine inflation, but expected inflation, given the exogenous path of nominal interest rates. (11.62) is consistent with any price level path that satisfies,

\[ p_{t+1} = p_t + i_t - r_t + \xi_{t+1} \]  

(11.63)

where \( \xi \) is any shock that satisfies \( E_t(\xi_{t+1}) = 0 \).

(11.63) suggests that there are multiple equilibria for the price level and inflation, depending on \( \xi \). This price level indeterminacy when the central bank follows an exogenous path for the nominal interest rate is also transferred to the money supply, through the money demand function (11.58). Consequently neither the money supply nor the price level can be determined uniquely when the central bank follows an exogenous path for the nominal interest rate.\(^9\)

Such an equilibrium is often referred to as a bubble or a sunspot equilibrium, since the equilibrium depends on external factors that have nothing to do with economic fundamentals.

However, not all interest rate rules result in price level indeterminacy. As suggested more than a century ago by Wicksell (1898), and we demonstrated in Chapter 10, if the central bank conditions its nominal interest rate on the price level, or inflation, then price level and inflation indeterminacy does not necessarily follow.

### 11.4.3 An Inflation Based Nominal Interest Rate Rule

Central banks predominantly use the nominal interest rate as their preferred monetary instrument. However, they follow policies according to which the path of nominal interest rates is not exogenous, but depends on past, current and expected future economic developments, mainly inflation. For example, if inflation rises, central banks usually raise nominal interest rates in order to reduce it, and vice versa. This was after all the essence of the Wicksell rule. Let us therefore assume the following rule for determining nominal interest rates,\(^10\)

\[ i_t = \rho + \phi \pi_t \]  

(11.64)

where \( \phi > 0 \) is the reaction of the central bank nominal interest rate to inflation.

From (11.59) και (11.64) we therefore have that,
\[ \pi_t = \frac{1}{\phi} E_t (\pi_{t+1}) + \frac{1}{\phi} (r_t - \rho) \quad (11.65) \]

where the real interest \( r \) depends only on real factors, as is the implication of “new” classical models.\(^\text{11}\)

Solving (11.65) under rational expectations,

\[ \pi_t = \sum_{s=0}^{\infty} \left( \frac{1}{\phi} \right)^s E_t (r_{t+s} - \rho), \text{ if } \phi > 1 \quad (11.66) \]

\[ \pi_{t+1} = \phi \pi_t - (r_t - \rho) + \xi_{t+1}, \text{ if } \phi \leq 1 \quad (11.67) \]

Thus, if the reaction of the central bank nominal interest rates to inflation is sufficiently pronounced (\( \phi > 1 \)), there is no indeterminacy problem for inflation. The fundamental solution is given by (11.66). If the reaction of the nominal interest rates to inflation is not sufficiently pronounced (\( \phi \leq 1 \)), then the problem of inflation indeterminacy and the possibility of “price bubbles” remains.

In any case, as we have already shown, in the “new” classical model of aggregate fluctuations only real factors affect fluctuations in real variables. Monetary factors and monetary policy only affect real money balances and nominal variables such as the price level and inflation, nominal interest rates and the nominal money stock.

### 11.5 Conclusions

“New” classical models of aggregate fluctuations imply that aggregate fluctuations are caused by real factors. This is why such models are often called real business cycle models.

New classical models are dynamic stochastic general equilibrium models (DSGE) based on optimizing behavior by both households and firms, flexible prices, and fully competitive markets. Households maximize their inter-temporal utility, firms maximize the present value of their profits, and markets function efficiently.

If the competitive general equilibrium models of this kind could explain all the features of aggregate fluctuations, then there would be no need for models that stress distortions in product and labor markets, and other market inefficiencies. However, “new” classical models have a number of weaknesses as models of aggregate fluctuations.

First, these models cannot account for the real effects of nominal and monetary shocks. For example, it is widely accepted that the Great Depression of the 1930s was caused by monetary and not real shocks. Similar views are prevalent regarding the recession of 2008-09, which was one of the deepest post World War II recessions.

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\(^{11}\) See for example, equation (11.57). We shall further assume that the process determining the real interest rate is stationary.
Second, even though “new” classical models can account for employment fluctuations, they only do so on the basis of inter-temporal substitution in labor supply. This explanation, however, is not sufficient to explain the phenomenon of high and persistent unemployment and the widely held view that unemployment is an involuntary condition for those who experience it, and not the result of a voluntary rational choice.

For these reasons, and despite the fact that “new” classical models are theoretically consistent, many economists consider them extreme as the sole explanation of aggregate fluctuations. The alternative class of models are “new keynesian” models, which assume that nominal wages and/or prices cannot adjust immediately in order to equilibrate labor and product markets. Thus, following nominal shocks, quantities have to adjust too, resulting in fluctuations in real variables, deviations of output and other real variables from their steady state values and involuntary unemployment. It is to these models that we now turn.
Annex to Chapter 11:  
A Log-Linear Approximation to the Stochastic Growth Model

This Annex sets out the stochastic growth model and derives an approximate analytical solution, based on a log-linear approximation around the steady state equilibrium. Following Campbell (1994), the model is thus transformed into a system of log-linear stochastic difference equations, which can be solved by the method of undetermined coefficients. We shall solve the model assuming that government expenditure is equal to zero.

The first equation of the model is the production function,

\[ Y_t = K^\alpha (A_t L_t)^{1-\alpha}, \quad 0<\alpha<1 \]  (A.11.1)

The second is the capital accumulation process,

\[ K_{t+1} = (1-\delta)K_t + Y_t - C_t \]  (A.11.2)

Finally there is a representative household which maximizes,

\[ U = E_t \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t \left( \ln C_t + b \ln (N_t - L_t) \right) \]  (A.11.3)

subject to the accumulation process (A.11.2).

Firms maximize profits, subject to the production function, and set the marginal product of capital and labor equal to the real interest rate and the real wage respectively. It thus follows that,

\[ w_t = (1-\alpha) \left( \frac{K_t}{A_t L_t} \right)^\alpha A_t \]  (A.11.4)

\[ r_t = \alpha \left( \frac{A_t L_t}{K_t} \right)^{1-\alpha} - \delta \]  (A.11.5)

From the first order conditions for the maximization of the utility of the representative household it also follows that,

\[ \frac{1}{C_t} = \frac{1+n}{1+\rho} E_t \left[ \frac{1}{C_{t+1}} (1 + r_{t+1}) \right] \]  (A.11.6)

\[ \frac{C_t}{N_t - L_t} = \frac{w_t}{b} \]  (A.11.7)

(A.11.6) is a stochastic version of the Euler equation for aggregate consumption, and (A.11.7) relates current consumption and leisure to the real wage.
A.11.1 Steady State

In the steady state, aggregate variables such as output, effective labor, capital and consumption grow at a rate $g+n$. Thus, from (A.11.6), the steady state real interest rate is determined by the condition,

$$1 + r = (1 + \rho)(1 + g)$$  \hspace{1cm} (A.11.8)

(A.11.8) implies that,

$$r = (1 + \rho)(1 + g) - 1 = \rho + g$$  \hspace{1cm} (A.11.9)

From the marginal productivity condition for capital (A.11.5), the steady state ratio of output to capital is determined by,

$$\frac{Y}{K} = \left(\frac{AL}{K}\right)^{\frac{1}{1-\alpha}} = \frac{r + \delta}{\alpha} = \frac{\rho + g + \delta}{\alpha}$$  \hspace{1cm} (A.11.10)

From (A.11.10), the steady state ratio of effective labor to capital is determined by,

$$\frac{AL}{K} = \left(\frac{\rho + g + \delta}{\alpha}\right)^{\frac{1}{1-\alpha}}$$  \hspace{1cm} (A.11.11)

Finally, from the capital accumulation process (A.11.2) and (A.11.10), the steady state consumption to output ratio is given by,

$$\frac{C}{Y} = 1 - \alpha \left(\frac{n + g + \delta}{\rho + g + \delta}\right)$$  \hspace{1cm} (A.11.12)

Note that the last term in (A.11.12) is the steady state savings rate.

A.11.2 Log-linearizing the Model around the Steady State

We shall consider fluctuations of the endogenous variables around the steady state. Outside the steady state the model is a system of non-linear equations in the logs of productivity, capital, labor, output and consumption. Non-linearities arise because of the depreciation rate, the equation for capital accumulation, the variable savings rate and the variable employment rate. Unlike the simplified model we examined in the text, we shall seek an approximate analytical solution, by taking a log-linear approximation of all equations around the steady state.

The Cobb Douglas production function can be log-linearized directly. From (A.11.1), it follows that,

$$y_t = \alpha k_t + (1 - \alpha)(a_t + l_t)$$  \hspace{1cm} (A.11.13)
where lowercase letters denote the difference of the log of the relevant variable from its steady state value.

The capital accumulation equation (A.11.2) is obviously not log-linear. Dividing by $K_t$, it can be written as,

$$\left(\frac{K_{t+1}}{K_t} - (1 - \delta)\right) = \frac{Y_t}{K_t} \left(1 - \frac{C_t}{Y_t}\right)$$  \hspace{1cm} (A.11.14)

Taking logs, (A.11.14) can be written as,

$$\ln[e^{\Delta k_t} - (1 - \delta)] = y_t - k_t + \ln[1 - e^{(e - \lambda_1 k_t)}]$$  \hspace{1cm} (A.11.15)

Taking a first order Taylor approximation of (A.11.15) around the steady state, and using the log-linear version of the production function (A.11.13), we end up with the following log-linear approximation of the accumulation equation around the steady state,

$$k_{t+1} = \lambda_1 k_t + \lambda_2 (a_t + l_t) + (1 - \lambda_1 - \lambda_2) c_t$$  \hspace{1cm} (A.11.16)

where,

$$\lambda_1 = \frac{1 + \rho + g}{1 + g}, \quad \lambda_2 = \frac{(1 - \alpha)(\rho + g + \delta)}{\alpha(1 + g)}.$$

We next turn to the determination of the real interest rate and the Euler equation for consumption.

From the marginal productivity condition for capital, (A.11.5), it follows that,

$$1 + r_{t+1} = \alpha \left(\frac{A_t L_{t+1}}{K_{t+1}}\right)^{1-\alpha} + (1 - \delta)$$  \hspace{1cm} (A.11.17)

Taking a log-linear approximation of (A.11.17) around the steady state, we get that,

$$r_{t+1} = \lambda_3 (a_{t+1} + l_{t+1} - k_{t+1})$$  \hspace{1cm} (A.11.18)

where,

$$\lambda_3 = \frac{(1 - \alpha)(\rho + g + \delta)}{1 + \rho + g}.$$

Substituting (A.11.18) in the Euler equation for consumption (A.11.6), and assuming the variables on the right hand side are jointly log-normal and homoskedastic, the Euler equation for consumption can be written as,

$$c_t = E_t c_{t+1} - E_t r_{t+1} = E_t c_{t+1} - \lambda_3 E_t (a_{t+1} + l_{t+1} - k_{t+1})$$  \hspace{1cm} (A.11.19)
Log-linearizing the marginal productivity condition for labor (A.11.4), and taking deviations from steady state, we get that deviations of the log of the real wage from steady state are given by,

\[ w_t = \alpha(k_t - l_t) + (1 - \alpha)a_t \]  \hspace{1cm} (A.11.20)

Finally, log-linearizing the first order condition for consumption and leisure, (A.11.7), using the marginal productivity condition for employment (A.11.20) to substitute for the logarithm of the real wage, we get,

\[ l_t = \nu\{\alpha k_t + (1 - \alpha)a_t - c_t\} \]  \hspace{1cm} (A.11.21)

where,

\[ \nu = \frac{1 - (\bar{L}/N)}{(\bar{L}/N) - \alpha(1 - (\bar{L}/N))} \]

\( \bar{L}/N \) is the steady state labor supply as a percentage of total available time. Following Prescott (1986) we shall assume that this is equal to one third.

In order to close the model, we need only specify the exogenous stochastic process driving productivity \( a \). We shall continue to assume, as in the main text, that it follows an AR(1) process of the form,

\[ a_t = \eta_a a_{t-1} + \varepsilon_t^A, \quad 0 < \eta_a < 1 \]  \hspace{1cm} (A.11.22)

**A.11.3 Solving the Model**

The model consists of equations (A.11.13), (A.11.16), (A.11.18), (A.11.19), (A.11.20), (A.11.21), and determines fluctuations around the steady state for output, the capital stock, consumption, employment, the real interest and the real wage. The exogenous shock driving the fluctuations is a productivity (technological) shock, that follows the AR(1) process in (A.11.22).

We can first solve the sub-system of (A.11.16), (A.11.19) and (A.11.21) for capital, employment and consumption, and then substitute in the other three equations to determine output, the real interest rate and the real wage.

The easiest way to solve the model analytically is to use the method of undetermined coefficients. We start from the equation for consumption, and conjecture that consumption will be a linear function of the two state variables \( k \) and \( a \), of the form,

\[ c_t = \eta_{ck}^t k_t + \eta_{ca}^t a_t \]  \hspace{1cm} (A.11.23)

where \( \eta_{ck} \) and \( \eta_{ca} \) are coefficients to be determined.

Substituting (A.11.23) in the employment equation (A.11.21), we get the solution for employment as,
where, $\eta_{LK} = \nu \alpha - \eta_{CK}$, and $\eta_{LA} = 1 - \alpha - \eta_{CA}$.

Substituting (A.11.23) and (A.11.24) in the capital accumulation equation (A.11.16), and making use of the exogenous process (A.11.22), we get the solution for the accumulation of capital as,

\[ (A.11.25) \]

Finally, we can substitute (A.11.24) and (A.11.25) in the Euler equation for consumption (A.11.19), and take the rational expectations solution, using the exogenous process (A.11.22) as well. We then find that,

\[ (A.11.26) \]

Comparing coefficients between (A.11.26) and (A.11.23), we can determine the undetermined coefficients $\eta_{CK}$ and $\eta_{CA}$.

**A.11.4 Aggregate Fluctuations around the Steady State.**

We can now use the solution we have obtained to characterize the fluctuations of the various aggregates around the steady state.

From (A.11.25) and (A.11.22), fluctuations in the capital stock are determined by,

\[ (A.11.27) \]

Fluctuations of the capital stock around its steady state value follow a stationary AR(2) process.

Substituting (A.11.27) and (A.11.22) in the consumption equation (A.11.23), we can see that fluctuations in consumption around its steady state follow a stationary ARMA(2,1) process of the form,

\[ (A.11.28) \]

Substituting (A.11.27) and (A.122) in the employment equation (A.11.24), we can see that fluctuations in employment around its steady state follow a stationary ARMA(2,1) process of the form,

\[ (A.11.29) \]
Finally, substituting (A.11.27) and (A.11.28) in the log-linear version of the aggregate production function (A.11.13), fluctuations of output around its steady state follow,

\[ y_t = (\eta_{kk} + \eta_A) y_{t-1} - \eta_{kk} \eta_{LA} y_{t-2} + (1-\alpha)(1+\eta_{LA}) \epsilon_t^A - (1-\alpha)(\eta_{kk} (1+\eta_{LA}) - \eta_{Lk} \eta_{KA}) \epsilon_{t-1} \]  \hspace{1cm} (A.11.30)

Thus, fluctuations of output around its steady state follow an ARMA(2,1) process as well.

In Figure 11.1 we present the results of a dynamic simulation of the model, for a 1% positive shock in productivity $a$. The parameter values we used in the simulation were $\alpha=0.333$, $\rho=0.02$, $g=0.02$, $\delta=0.03$, $\nu=2$, $\eta_A=0.90$.

As can be seen from the simulations, all real variables move pro-cyclically, as innovations in productivity affect output, the capital stock, consumption and employment in the same direction. Real wages and the real interest rate also move pro-cyclically. Gradually, all variables converge to the steady state unless the system is disturbed by another shock.
Figure 11.1
Dynamic Simulation of the Stochastic Growth Model following a 1% persistent shock to productivity
References