Chapter 13
A “New Keynesian” Model with Periodic Wage Contracts

The realization of the instability of the original Phillips curve has gradually led to a paradigm shift in macroeconomics relative to the standard Keynesian model that we examined in the previous chapter.

Since the 1970s, the macroeconomics of aggregate fluctuations has been emphasizing the microeconomic foundations of all behavioral relations, and in particular the consumption and investment functions and the short-term determination of wages, prices and the equilibrium unemployment rate. In addition, the “rational expectations” hypothesis, which requires that households and firms form their expectations about future variables, taking into account the actual process determining the evolution of these variables, has become the dominant expectations hypothesis. The hypothesis of adaptive expectations, was gradually abandoned. Thus, macroeconomic models of aggregate fluctuations gradually evolved into dynamic stochastic general equilibrium models based on rational expectations.

The “new classical” model we examined in Chapter 11 is an example of such a model, in which wages and prices are perfectly flexible and equilibrate both the product and labor markets. In “new classical” models, only real shocks, such as shocks to productivity, can affect the fluctuations of output, employment and other real variables. Monetary shocks only affect nominal variables, such as the price level and inflation. In addition, employment fluctuations are based on inter-temporal substitution and, thus, there is no involuntary unemployment in the “new classical” model.

The short run neutrality of money implied by “new classical” models was initially troublesome for their proponents, as these models were not compatible with the existence of a positive short run relation between inflation and employment, as suggested by the expectations augmented Phillips curve. Lucas (1972, 1973), developed a “new classical model” which was consistent with a positive short run relation between inflation and employment. This model, which was subsequently implemented empirically by Sargent (1973, 1976), was based on the assumption that firms did not have full information about the price level at the time they made their production decisions, and they attributed part of any change in the price level to a change in the relative price of their product. Thus, when inflation was unexpectedly high, all producers thought the relative price of their output had gone up, and thus increased production and employment. The opposite happened when inflation was unexpectedly low.¹

¹ A log linear version of the Lucas (1972) model was analyzed by Barro (1976), and was extended to incorporate the labor market and inter-temporal substitution in labor supply by Alogoskoufis (1983). In the extended model, as workers could not observe the price level immediately, they systematically attributed part of unexpected inflation to relative price changes, and a temporary increase in their real wage. Thus, they increased labor supply in response to the increased labor demand of firms, and employment and output rose in response to unanticipated inflation. However, there was no involuntary unemployment and fluctuations in employment were based on inter-temporal substitution in labor supply.
However, this “new classical” explanation of the short run relation between inflation and output and employment was still not compatible with involuntary unemployment, and could only account for temporary deviations of output and employment from their “natural levels”, due to inter-temporal substitution in labor supply and unanticipated inflation.

An alternative approach, due to Gray (1976), Fischer (1977) and Taylor (1979), emphasized periodic nominal wage contracts. This approach descended directly from the General Theory, treating nominal wages as temporarily fixed. In the Gray-Fischer model, nominal wage contracts are assumed to be negotiated at the beginning of every period, or at the beginning of alternate periods. In addition, nominal wages are assumed to remain fixed for the duration of the contract. Thus, nominal wages depend on prior expectations about the evolution of the price level, productivity and all other shocks. If inflation turns out to be higher than expected, then real wages fall, firms demand more labor, and employment rises. The opposite happens when inflation turns out to be lower than expected. Thus, these models have keynesian features, and have formed the basis of the so called new keynesian approach to aggregate fluctuations.

In this chapter we analyze a “new Keynesian” model based on such periodic nominal wage contracts, which is comparable to the “new classical” model without capital. It not only allows for nominal shocks and monetary policy to affect the fluctuations of real variables, but it also allows for the existence of “involuntary” unemployment. The model builds on one of the key insights of the General Theory, namely the short run rigidity of nominal wages, as envisaged by Gray and Fischer contracts. In all other respects it is based on inter-temporal optimization on the part of both households and firms.

The model is a dynamic, stochastic general equilibrium model, in which non indexed nominal wage contracts are negotiated periodically by “insiders” in the labor market. There are two distortions in the model of this chapter compared to the “new classical” model without capital that we analyzed in Chapter 11. The first is a real distortion, arising from the fact that “outsiders” are disenfranchised from the labor market. As a result, wage contracts do not seek to maintain full employment and there is a positive “natural” rate of unemployment. The second is a nominal distortion, arising from the fact that nominal wage contracts are not indexed, and can only be reopened at the beginning of each period, before the realization of current nominal and real shocks. Thus, nominal wages are set on the basis of prior rational expectations about the various unobserved shocks.

The real distortion in our model makes the “natural” rate of unemployment inefficiently high, while the nominal distortion allows for nominal shocks to have temporary real effects. Thus, nominal shocks and, by extension, monetary policy, are able to affect fluctuations in both inflation and real variables such as output, employment, unemployment, real wages and the real interest rate.

The model is characterized by an expectations augmented “Phillips curve”, in which deviations of output and employment from their “natural” level depend on unanticipated current inflation, and unanticipated productivity shocks, which affect the relation between real wages and productivity.

Aggregate demand is determined by the optimal behavior of a representative household, with access to a competitive financial market, choosing the path of consumption and real money balances in order to maximize its inter-temporal utility function. Thus, both the consumption function and the money demand function are derived from inter-temporal microeconomic foundations. The model is
also characterized by exogenous shocks to productivity, preferences for consumption and money demand, as well as labor market shocks.\(^2\)

Thus, the model is in essence a dynamic stochastic model that incorporates many of the features of the \(AS-AD\) version of the Keynesian model that we presented in the previous chapter.

We analyze aggregate fluctuations in this model under two alternative monetary regimes. The first is an exogenous process for the rate of growth of the money supply and the second is a feedback interest rate rule, according to which the nominal interest rate responds to deviations of inflation from the target of the central bank, and deviations of output from its “natural” level.

Contrary to the “new classical” model, monetary shocks affect real variables in this model, causing temporary deviations of output, employment, unemployment, real wages and the real interest rate from their “natural” levels. The exact variance of such deviations depends on the monetary rule. Under an exogenous process for the rate of growth of the money supply, all shocks affect aggregate fluctuations. Under a feedback interest rate rule, only productivity and nominal interest rate shocks affect aggregate fluctuations. We thus demonstrate the dependence of aggregate fluctuations not only on exogenous shocks, but on the form of the monetary policy rule followed by the central bank.

We also extend the model to account for persistence in deviations of unemployment and output from their “natural” levels. The extension is based on a dynamic model of the “Phillips Curve”, in which unanticipated shocks to inflation and productivity have persistent effects on unemployment, and these persistent effects are compatible with full inter-temporal optimization on the part of labor market “insiders”. The propagation mechanism that causes unanticipated nominal and real shocks to produce persistent deviations of unemployment and output from their “natural” rate is the partial adjustment of labor market insiders to employment shocks. We demonstrate that under a Taylor rule, the only shocks that cannot be completely neutralized by monetary policy are productivity shocks and, of course, monetary policy shocks. Fluctuations of deviations of unemployment and output from their “natural” rates display persistence and are driven by these two types of shocks. Because of the endogenous persistence of deviations of unemployment from its “natural” rate, the equilibrium inflation rate also displays persistence around the inflation target of the central bank.

13.1 “Insiders”, Wage Setting and the “Phillips Curve”

The wage setting model introduced in this chapter combines and extends two strands of the literature.

The first strand of the literature is the insider-outsider theory of wage determination of Lindbeck and Snower (1986), Blanchard and Summers (1986) and Gottfries (1992). According to this approach, there is an asymmetry in the wage setting process between “insiders”, who already have jobs, and “outsiders” who are seeking employment. “Outsiders” are disenfranchised from the labor market, and wages are set by “insiders”, who seek to maximize the real wage consistent with their own employment, and not with the employment of the full labor force. This causes the “natural” rate of unemployment to be inefficiently high. The total number of “insiders” in the economy is

\(^2\) Labor market shocks take the form of shocks to the number of “insiders”, and are similar to labor supply shocks in a competitive labor market model.
assumed to be subject to stochastic shocks, resulting in exogenous fluctuations in the “natural” rate of unemployment and the “natural” level of employment and output.

Second, the model incorporates the Gray (1976)-Fischer (1977) model of predetermined nominal wages, according to which nominal wage contracts are negotiated at the beginning of each period, and wages remain fixed for one period. Because current shocks, including current inflation, are not known when nominal wage contracts are negotiated, unanticipated inflation reduces real wages and causes employment to increase along a downward sloping labor demand curve. Thus, the model produces a positive short run relation between unanticipated inflation and output and employment, i.e an expectations augmented Phillips curve.

Employment and output are determined by competitive firms, which set employment in each period at the level which equates the real wage to the marginal product of labor. The marginal product of labor is subject to persistent productivity shocks, which affect both labor demand, and the output produced for given employment.

13.1.1 Output, Employment and Labor Demand

Consider an economy consisting of competitive firms, indexed by $i$, where $i \in \{0,1\}$. Labor is the only variable factor of production, and firms determine employment by equating the marginal product of labor to the real wage.

The production function of firm $i$ is given by,

$$Y(i) = A_t L(i)^{1-\alpha}$$

where $Y(i)$ is output, $A$ is exogenous productivity, and $L(i)$ is employment. $t$ is a time index, where $t=0,1, \ldots$ $0<1-\alpha<1$ is the elasticity of output with respect to employment.

Employment is determined by firms, which maximize profits by equating the marginal product of labor to the real wage. Thus, employment is determined by the condition that,

$$(1-\alpha)A_t L(i)^{\alpha} = \frac{W(i)}{P_t}$$

where $W(i)$ is the nominal wage of firm $i$, and $P$ is the price for the product of firm $i$. Since the product market is assumed to be competitive, all firms face the same price, and $P(i)=P$ for all firms.

In log-linear form, (13.1) and (13.2) can be written as,

$$y(i)_t = a_t + (1-\alpha)l(i)_t$$

$$l(i)_t = i^\frac{1}{\alpha}(w(i)_t - p_t - a_t)$$

where $i = \frac{\ln(1-\alpha)}{\alpha}$
Lowercase letters denote the logarithms of the corresponding uppercase variables. (3) determines output as a positive function of employment, and (4) determines employment as a negative function of deviations of real wages from productivity.

### 13.1.2 Wage Setting and Employment in an “Insider Outsider” Model

Nominal wages are set by “insiders” in each firm, at the beginning of each period, before variables, such as current productivity and the current price level are known. Thus, nominal wages are set on the basis of the rational expectations of “insiders” about these shocks. Nominal wages remain constant for one period, and they are reset at the beginning of the following period.

Thus, this model is characterized by the real distortions emphasized by Lindbeck and Snower (1976), leading to an inefficiently high “natural” rate of unemployment, and by the nominal wage stickiness of the Gray (1976), Fischer (1977), Gottfries (1992) models. Employment is determined ex post by firms, given the contract wage, after the current price level and productivity have been revealed. This set up leads to temporary real effects of nominal shocks and monetary policy.

The number of “insiders”, who at the beginning of each period determine the contract wage, is assumed exogenous. The key objective of “insiders” is to set a nominal wage which, given their rational expectations about the price level and productivity, will minimize deviations of expected employment from an employment target determined by “insiders” in each firm.

The expectations on the basis of which wages are set depend on information available until the end of period $t-1$, but not on information about prices and productivity in period $t$.

On the basis of the above, we assume that the objective of wage setters in each firm is to make expected employment satisfy a path that minimizes the following quadratic inter-temporal loss function,

$$
\min_{E_t} \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{2} \left( l(i)_{t+1} - \bar{n}(i)_{t+1} \right)^2 \right] 
$$

(13.5)

$\bar{n}$ is the logarithm of the number of “insiders” in each firm. $\beta=1/(1+\rho)<1$ is the discount factor, with $\rho$ being the pure rate of time preference. (5) is minimized subject to the labor demand equation (13.4).

We assume that the total number of “insiders” in the economy is always strictly smaller than the labor force. We thus assume that,

$$
\int_{i=0}^{1} \tilde{n}(i) \, di = \tilde{n}, \quad n_t < \tilde{n}, \quad \forall t 
$$

(13.6)

where $n$ is the log of the labor force.

From the first order conditions for a minimum of (13.5), wages are set so that expected employment for each firm satisfies,
Integrating over $i$, expected aggregate employment must then satisfy,

$$E_t[l(i)] = \tilde{n}(i) \quad (13.7)$$

Integrating over $i$, expected aggregate employment must then satisfy,

$$E_t[l_t] = n_t \quad (13.8)$$

$(13.8)$ is the same as $(13.7)$ without the $i$ index. $(13.8)$ determines the “natural” level of employment, solely on the basis of the number of “insiders” in the wage setting process. Since the number of insiders is assumed to always be smaller than the labor force, the “natural” level of employment is inefficiently low.

Actual employment is determined by firms, after the nominal wage has been set, and after information about current prices, productivity and other shocks has been revealed.

Integrating the labour demand function over the number of firms $i$, aggregate employment is given by,

$$l_t = \tilde{l} - \frac{1}{\alpha}(w_t - p_t - a_t) \quad (13.9)$$

From $(13.8)$ and $(13.9)$, the contract wage satisfies,

$$w_t = E_{t-1} p_t + E_{t-1} a_t - \alpha (\tilde{n}_t - \tilde{l}) \quad (13.10)$$

The wage is set so as to make expected employment equal to the number of insiders, and is based on one period ahead expectations about the price level and productivity.

### 13.1.3 An Expectations Augmented Phillips Curve

Substituting $(13.10)$ in $(13.9)$, actual employment evolves according to,

$$l_t = \tilde{l} + \frac{1}{\alpha} \left( p_t - E_{t-1} p_t + a_t - E_{t-1} a_t \right) = \tilde{n}_t + \frac{1}{\alpha} \left( \pi_t - E_{t-1} \pi_t + a_t - E_{t-1} a_t \right) \quad (13.11)$$

where, $\pi_t = p_t - p_{t-1}$ is the rate of inflation.

From $(13.11)$, employment deviates from its “natural” level to the extent that there are unanticipated shocks to inflation and productivity. Unanticipated increases in inflation cause a reduction in real wages and increase labour demand and employment, while, unanticipated increases in productivity increase productivity relative to real wages, and thus also increase labour demand and employment.

We can define the unemployment rate as,

$$u_t = n_t - l_t \quad (13.12)$$
We can define the “natural rate” of unemployment as,

\[ \bar{u}_t \approx n_t - \bar{n}_t \]  

(13.13)

From (11), (12) and (13), it follows that,

\[ u_t = \bar{u}_t - \frac{1}{\alpha} \left( \pi_t - E_{t-1} \pi_t + a_t - E_{t-1} a_t \right) \]  

(13.14)

The unemployment rate deviates from its “natural” rate as a result of unexpected shocks to inflation and productivity, because both reduce real wages relative to productivity, compared with the prior expectations of wage setters. (13.14) has the form of an expectations augmented Phillips curve, which arises because nominal wages are set for one period and before current inflation and productivity are known.

We can also express this expectations augmented Phillips curve in terms of output. From the log-linear version of the firm production function in (13.3), aggregating over firms, we get an aggregate production function in log-linear form, as,

\[ y_t = a_t + (1 - \alpha) l_t \]  

(13.15)

Substituting (13.11) in the log-linear version of the production function (13.15), output supply evolves according to,

\[ y_t = y_\tau + \frac{1 - \alpha}{\alpha} \left( \pi_t - E_{t-1} \pi_t + a_t - E_{t-1} a_t \right) \]  

(13.16)

where,

\[ y_\tau = (1 - \alpha) n_\tau + a_\tau \]  

(13.17)

is the “natural” level of output.

Unexpected shocks to inflation and productivity cause output to be higher than its “natural” level, as they cause employment to be higher than its own “natural” level. (13.16) can be seen as the output version of the “expectations augmented Phillips curve”, or as a short run “output supply function”.

It is worth noting that, if wage setting did not take place in advance but during the actual period, there would be no real effects from unexpected inflation and no employment effects from shocks to productivity. Essentially, we would have a “quasi” classical model, with a positive “natural rate” of unemployment and no “Phillips curve”. Thus, the assumption that nominal wages are set in advance, and before current shocks to inflation and productivity become known to wage setters, is very important for the properties of this model.

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3 The concept of the “natural” rate of unemployment is due to Friedman (1968) and was analyzed in the context of an expectations augmented Phillips curve in Chapter 12.
13.1.4 The “Natural” Rate of Unemployment and the “Natural” Level of Output

It is worth distinguishing between the “natural” level of output and the “full employment” level output. Full employment output is given by,

\[ y^f_t = (1 - \alpha) n_t + a_t \]  

(13.18)

Full employment output is always higher than the “natural” level of output in this model. The reason is that equilibrium employment is lower than full employment, since the pool of “insiders”, who are ones who determine equilibrium employment through their wage setting behavior, is smaller than the labor force. Thus, because of this real distortion in the labor market, the “natural” level of output is inefficiently low, and the “natural rate” of unemployment is inefficiently high.

From (13.17) and (13.18), the relation between the “natural rate” of unemployment and deviations of output from “full employment” output is given by,

\[ y^f_t - y_t = (1 - \alpha) (n_t - \bar{n}_t) = (1 - \alpha) \bar{u}_t \]  

(13.19)

This is the real distortion in this model. Because of the inefficiency in the labor market, due to the market power of “insiders”, the equilibrium level (“natural level”) of employment is lower than full employment, the “natural level” of output is lower than full employment output, and the “natural rate” of unemployment is positive. Furthermore, equilibrium unemployment is involuntary, as the unemployed outsiders would be prepared to work at the prevailing real wage.

13.2. The Determination of Aggregate Consumption and Money Demand

We next turn to the determination of aggregate demand. We assume that the economy consists of a large number of identical households \( j \), where \( j \in [0, 1] \). Each household member supplies one unit of labor, and unemployment impacts all households in the same manner. Thus, if \( H \) is the number of households and \( N \) is the aggregate labor force, each household has \( N/H \) members. Of those, some are “insiders” in the labor market, and the rest are “outsiders”. The proportion of insiders is the same for all households. In addition, the proportion of the unemployed is also assumed to be the same for all households.

The representative household chooses (aggregate) consumption and real money balances in order to maximize,

\[ E_t \sum_{s=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^s \left( \frac{1}{1 - \theta} \left( V^C_{t+s} C_{t+s}^{1-\theta} + V^M_{t+s} \left( \frac{M}{P} \right)^{1-\theta} \right) \right) \]  

(13.20)

subject to the sequence of expected budget constraints,

\[ E_t \left( F_{t+s+1} - (1 + i_{t+s}) \left( F_{t+s} - \frac{i_{t+s}}{1 + i_{t+s}} M_{t+s} + P_{t+s} (Y_{t+s} - C_{t+s} - T_{t+s}) \right) \right) = 0 \]  

(13.21)

where \( F_t = B_t + M_t \).
\(\rho\) denotes the pure rate of time preference, \(\theta\) is the inverse of the elasticity of inter-temporal substitution, \(i\) the nominal interest rate, \(F\) the current value of the financial assets of the household (one period nominal bonds \(B\) and money \(M\)), \(Y\) real non interest income and \(T\) real taxes net of transfers. \(V^C\) and \(V^M\) denote exogenous stochastic shocks in the utility from consumption and real money balances respectively.

From the first order conditions for a maximum,

\[
V^C_i C_i^{-\theta} = \lambda_i (1 + i_i) P_t \tag{13.22}
\]

\[
V^M_i \left( \frac{M}{P} \right)_t^{-\theta} = \lambda_i i_i P_t \tag{13.23}
\]

\[
E_t \lambda_{t+1} = E_t \left( \frac{1 + \rho}{1 + i_{t+1}} \right) \lambda_t \tag{13.24}
\]

where \(\lambda_t\) is the Lagrange multiplier in period \(t\).

(13.22)-(13.24) have the standard interpretations. (13.22) suggests that at the optimum the household equates the marginal utility of consumption to the value of savings. (13.23) suggests that the household equates the marginal utility of real money balances to the opportunity cost of money. Finally, (13.24) suggests that at the optimum, the real interest rate, adjusted for the expected increase in the marginal utility of consumption, is equal to the pure rate of time preference.

From (13.22), (13.23) and (13.24), eliminating \(\lambda\),

\[
\left( \frac{M}{P} \right)_t = C_t \left( \frac{V^C_i}{V^M_i} \right)^{\frac{1}{\theta}} \tag{13.25}
\]

\[
E_t \left( \frac{V^C_i (C_{t+1})^{-\theta}}{P_{t+1}} \right) = \left( \frac{1 + \rho}{1 + i_t} \right) \left( \frac{V^C_i (C_i)^{-\theta}}{P_t} \right) \tag{13.26}
\]

(13.25) is the money demand function, which is proportional to consumption and a negative function of the nominal interest rate, and (13.26) is the familiar Euler equation for consumption.

Log-linearizing (13.25) and (13.26),

\[
m_t - p_t = c_t - \frac{1}{\theta} \ln \left( \frac{i_t}{1 + i_t} \right) + \frac{1}{\theta} (v^M_t - v^C_t) \tag{13.27}
\]

\[
c_t = E_t c_{t+1} - \frac{1}{\theta} (i_t - E_t \pi_{t+1} - \rho) + \frac{1}{\theta} (v^C_t - E_t v^C_{t+1}) \tag{13.28}
\]
where lowercase letters denote natural logarithms, and, \( \pi_t = p_t - p_{t-1} \) is the rate of inflation.\(^4\)

We then turn to the determination of equilibrium in the product and money markets.

**13.3 Equilibrium in the Product and Money Markets**

Since there is no capital and investment in this model, and no government expenditure, product market equilibrium implies that output is equal to consumption.

\[ Y_t = C_t \tag{13.29} \]

Substituting (13.29) in (13.27) and (13.28), we get the money and product market equilibrium conditions,

\[ m_t - p_t = y_t - \frac{1}{\theta} \ln \left( \frac{i_t}{1+i_t} \right) + \frac{1}{\theta} \left( v_t^M - v_t^C \right) \tag{13.30} \]

\[ y_t = E_t y_{t+1} - \frac{1}{\theta} \left( i_t - E_t \pi_{t+1} - \rho \right) + \frac{1}{\theta} \left( v_t^C - E_t v_{t+1}^C \right) \tag{13.31} \]

(13.30) is the money market equilibrium condition, the equivalent of the *LM Curve* in the traditional Keynesian model, and (13.31) is the product market equilibrium condition, the equivalent of the *IS Curve*. (13.31) is often referred to as the *new keynesian IS curve*.

Since output demand depends on deviations of the real interest from the pure rate of time preference, the real interest rate is the relative price that adjusts to equilibrate output demand with output supply. No other relative price can play this role, as the real wage is determined in order to make expected labor demand equal to the number of “insiders” in the labor market.

**13.3.1 The “Natural” and the Current Real Interest Rate**

The real interest rate is defined by the Fisher (1896) equation,\(^5\)

\[ r_t = i_t - E_t \pi_{t+1} \tag{13.32} \]

The “natural” real interest rate is determined by the product market equilibrium condition, when output is at its “natural” level. From (13.17) and (13.31), the “natural” real interest rate is given by,

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\(^4\)Technically, since the logarithm of the expectation of a product (or ratio) of two random variables is not equal to the sum (or difference) of the expectations of the logarithms of the relevant random variables, (13.28) must also contain second order terms, depending on the covariance matrix of consumption, inflation and shocks to preferences for consumption and inflation. Assuming that all exogenous shocks are stationary stochastic processes, these second order terms are constant and can be ignored.

\(^5\)To quote from Fisher (1896), “When prices are rising or falling, money is depreciating or appreciating relative to commodities. Our theory would therefore require high or low interest according as prices are rising or falling, provided we assume that the rate of interest in the commodity standard should not vary.” (p. 58). The rate of interest in the commodity standard is the real interest rate, and rising or falling prices are expected inflation. The Fisher equation was further elaborated in Fisher (1930), where it was made even clearer that Fisher referred to expected inflation.
The “natural” real interest rate is equal to the pure rate of time preference, but also depends positively on deviations of current shocks to consumption from anticipated future shocks, and negatively on deviations of current productivity shocks from anticipated future shocks, as well as deviations of the current “natural” level of employment from its anticipated future level.

Thus, real shocks that cause a temporary increase in the “natural” level of output reduce the “natural” real rate of interest, in order to bring about a corresponding reduction in consumption and maintain product market equilibrium. On the other hand, real shocks that cause a temporary increase in consumption, require an increase in the “natural” real rate of interest, in order to induce lower consumption, and maintain product market equilibrium.

Because of the nominal rigidity of wages for one period, the current equilibrium real interest deviates from its “natural” rate. The current real interest rate is determined by the equation of the output demand function (13.31) with the output supply function (13.16). It is thus determined as,

\[
    \tilde{r}_t = \rho - \theta \left( (1 - \alpha) \left( \tilde{n}_t - \bar{n}_{t+1} \right) + (a_t - E_t a_{t+1}) \right) + \left( v^c_t - E_t v^c_{t+1} \right)
\]

The “natural” real interest rate is equal to the pure rate of time preference, but also depends positively on deviations of current shocks to consumption from anticipated future shocks, and negatively on deviations of current productivity shocks from anticipated future shocks, as well as deviations of the current “natural” level of employment from its anticipated future level.

\[
    \tilde{r}_t = \rho - \theta \left( (1 - \alpha) \left( \tilde{n}_t - \bar{n}_{t+1} \right) + (a_t - E_t a_{t+1}) \right) + \left( v^c_t - E_t v^c_{t+1} \right)
\]

Unanticipated shocks to inflation or productivity, which cause a temporary rise in current output relative to its “natural” level, also reduce the current real interest rate relative to its “natural” rate. This is the “Wicksellian” mechanism in this model. We shall return to this mechanism when we discuss alternative interest rate rules.

### 13.3.2 Equilibrium Fluctuations with Exogenous Real Shocks

In what follows, we shall assume that the logarithms of the exogenous shocks to preferences and productivity follow stationary AR(1) processes.

\[
    v^C_t = \eta_C v^C_{t-1} + \epsilon^C_t
\]

\[
    v^M_t = \eta_M v^M_{t-1} + \epsilon^M_t
\]

\[
    a_t = \eta_A a_{t-1} + \epsilon^A_t
\]

where the autoregressive parameters satisfy, \( 0 < \eta_C, \eta_M, \eta_A < 1 \), and \( \epsilon^C, \epsilon^M, \epsilon^A \), are white noise processes.

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6 The concept of a “natural” rate of interest was introduced by Wicksell (1898). To quote, “There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods. It comes to much the same thing to describe it as the current value of the natural rate of interest …” (p. 102). It is worth noting that Friedman (1968) defined the “natural” rate of unemployment in direct analogy to the definition of the “natural” rate of interest by Wicksell (1898). He made a direct reference to Wicksell, and was fully aware of the analogy between the two concepts.
We shall further assume that the (log of the) labor force is fixed at \( n \), and that the exogenous number of “insiders” also follows a stationary AR(1) process, of the form,

\[
\ddot{n}_t = (1 - \eta_N) \dot{n} + \eta_N \ddot{n}_{t-1} + \epsilon^N_t
\]  

(13.38)

where \( 0 < \ddot{n} < n \), is a constant, \( 0 < \eta_N < 1 \), and \( \epsilon^N \) is a white noise process. \( \epsilon^N \) is a labor market shock, increasing the number of “insiders”.

From (13.38), and the definition of the “natural” rate of unemployment in (13.13), we also have that,

\[
\ddot{u}_t = (1 - \eta_N) \dot{u} + \eta_N \ddot{u}_{t-1} - \epsilon^N_t
\]  

(13.39)

Thus, the “natural” rate of unemployment converges to a constant, but it displays fluctuations around this constant, caused by persistent shocks to the number of insiders.

With these assumptions, current employment, unemployment, output, real wages and the real interest rate, as functions of the exogenous shocks and shocks to inflation, evolve according to,

\[
l_t = \ddot{n}_t + \frac{1}{\alpha} \left( \pi_t - E_{t-1} \pi_t + \epsilon^A_t \right)
\]  

(13.40)

where \( \ddot{n}_t \) is given by (13.38).

\[
u_t = \ddot{u}_t + \frac{1}{\alpha} \left( \pi_t - E_{t-1} \pi_t + \epsilon^A_t \right)
\]  

(13.41)

where \( \ddot{u}_t \) is given by (13.39).

\[
y_t = y_t + \frac{1 - \alpha}{\alpha} \left( \pi_t - E_{t-1} \pi_t + \epsilon^A_t \right)
\]  

(13.42)

where, \( y_t = (1 - \alpha) \ddot{n}_t + \ddot{u}_t \).

\[
w_t - p_t = (w - p)_t - \left( \pi_t - E_{t-1} \pi_t \right)
\]  

(13.43)

where, \( (w - p)_t = \eta_A a_{t-1} - \alpha (\ddot{n}_t - \ddot{u}_t) \).

\[
r_t = r_t - \frac{\theta(1 - \alpha)}{\alpha} \left( \pi_t - E_{t-1} \pi_t + \epsilon^A_t \right)
\]  

(13.44)

where \( r_t = \ddot{r}_t - \theta \left( (1 - \alpha)(1 - \eta_N)(\ddot{n}_t - \ddot{n}) + (1 - \eta_A) a_t \right) + (1 - \eta_C) \nu^C_t \).

The “natural” rates (or levels) of real variables evolve as functions of the exogenous real shocks. In the absence of the nominal rigidity due to the assumption that nominal wages are set in advance and remain fixed for one period, the evolution of real variables would be equal to their “natural” levels. The model would in all respects be similar to a “new classical” model.
However, unanticipated inflation, and innovations in productivity, by reducing real wages relative to productivity, cause a temporary increase in employment and output above their “natural” level, and a temporary reduction in unemployment and the real interest rate below their “natural” rates. Since inflation is also affected by nominal shocks, unanticipated nominal shocks have real effects in this model.

13.4 Aggregate Fluctuations under an Exogenous Money Supply Rule

In order to close the model, we must make assumptions about the evolution of nominal variables such as the money supply. We shall initially assume that the money supply follows a random walk with drift, of the form,

\[ m_t = \mu + m_{t-1} + \varepsilon_t^S \] (13.45)

where \( \mu \) is a constant and \( \varepsilon_t^S \) a white noise shock to the money supply.

(13.45) can be viewed as a stochastic constant growth rule for the money supply, followed by the central bank. For example, Friedman (1960) proposed such a rule for monetary policy.

With this assumption, the steady state rate of growth of the money supply is equal to \( \mu \), and since growth is equal to zero in this model, steady state inflation is also equal to \( \mu \), and the steady state nominal interest rate is equal to \( \rho + \mu \).

The money demand function (13.30) can be approximated around the steady state nominal interest rate \( \rho + \mu \) as,

\[ m_t - p_t = y_t \left( 1 - \frac{1}{\theta} \ln \left( \frac{1 + i_t}{1 + i_t} \right) \right) + \frac{1}{\theta} \left( v_t^M - v_t^C \right) = m_0 + y_t - \zeta (r_t + E_t \pi_t) + \frac{1}{\theta} \left( v_t^M - v_t^C \right) \] (13.46)

where \( m_0 = -\frac{1}{\theta} \left( \ln \left( \frac{\rho + \mu}{1 + \rho + \mu} \right) \right) - \frac{1}{\theta} \left( \frac{1 + \rho + \mu}{1 + \rho + \mu} \right) \), and, \( \zeta = \frac{1}{\theta (\rho + \mu)(1 + \rho + \mu)} > 0 \).

\( \zeta \) is the semi-elasticity of money demand with respect to the nominal interest rate.

Substituting for real output and the real interest rate from (13.42) and (13.44) and solving for the price level, we get that,

\[ p_t \left( 1 + \zeta + \frac{(1 + \zeta \theta)(1 - \alpha)}{\alpha} \right) - E_{t-1} \rho_t \left( \frac{(1 + \zeta \theta)(1 - \alpha)}{\alpha} \right) - \zeta E_t p_{t+1} = z_t \] (13.47)

where \( z_t = m_t - \tilde{y}_t + \zeta \tilde{r}_t - \left( \frac{(1 + \zeta \theta)(1 - \alpha)}{\alpha} \right) \tilde{E}_t^A - \frac{1}{\theta} \left( v_t^M - v_t^C \right) - m_0 \)

13.4.1 The Real Effects of Monetary Shocks
In order to solve for the price level and inflation, we shall first abstract from real shocks and assume that the only shocks are monetary shocks, i.e. shocks to the money supply process \( \varepsilon^S \) and shocks to money demand \( \varepsilon^M \). Monetary shocks do not affect the “natural” level of output or the “natural” real rate of interest, so we can normalize them to zero.

In the presence of monetary shocks, the rational expectations solution of (13.47) is given by,

\[
(13.48)
\]

From (13.48), unanticipated inflation is given by,

\[
(13.49)
\]

Substituting (13.49) in (13.42), we get that real output is determined by,

\[
(13.50)
\]

Purely monetary shocks, such as shocks to the money supply and money demand, by causing unanticipated changes in inflation, cause temporary deviations of output from its “natural” level. The reason is that nominal wages are predetermined, based on expected inflation at the beginning of each period. By causing unanticipated changes in inflation, monetary shocks affect real wages and employment, output and the real interest rate.

Thus, in this model, because of the nominal distortion of predetermined nominal wages, purely monetary shocks have temporary real effects on output, employment, unemployment, real wages and the real interest rate.

13.4.2 The Effects of Real and Monetary Shocks on Prices and Output

Real shocks would also affect inflation, unanticipated inflation and fluctuations in real variables through \( z_t \) in (13.47). Real shocks in this model affect both the “natural” level of output and the real interest rate, and deviations of output and the real interest rate from their “natural” levels, either through unanticipated inflation, or, in the case of productivity shocks, directly.

We can use (13.47) to solve for the price level in terms of all the shocks. The solution takes the following form.

\[
(13.51)
\]

where,
\[ \ddot{p} = \mu - m_0 - (1 - \alpha) \dot{n} + \zeta \rho \]  
(13.52 a)

\[ \chi_A = \frac{1 + \zeta \theta (1 - \eta_A)}{1 + \zeta (1 - \eta_A)} \eta_A \]  
(13.52 b)

\[ \chi_C = \frac{1 + \zeta \theta (1 - \eta_C)}{1 + \zeta (1 - \eta_C)} \frac{\eta_C}{\theta} \]  
(13.52 c)

\[ \chi_M = -\frac{1}{1 + \zeta (1 - \eta_M)} \frac{\eta_M}{\theta} \]  
(13.52 d)

\[ \chi_N = -\frac{1 + \zeta \theta (1 - \eta_N)}{1 + \zeta (1 - \eta_N)} (1 - \alpha) \]  
(13.52 e)

\[ \psi_A = -\left( \frac{(1 + \zeta \theta)(1 - \alpha)}{\alpha} + \frac{\alpha (1 + \zeta \theta (1 - \eta_A))}{\alpha (1 + \zeta (1 - \eta_A)) + (1 + \zeta \theta)(1 - \alpha)} \right) \]  
(13.52 f)

\[ \psi_C = \frac{\alpha (1 + \zeta \theta (1 - \eta_C))}{\alpha (1 + \zeta (1 - \eta_C)) + (1 + \zeta \theta)(1 - \alpha) \theta} \]  
(13.52 g)

\[ \psi_S = \frac{\alpha}{\alpha + (1 - \alpha)(1 + \zeta \theta)} \]  
(13.52 h)

\[ \psi_M = -\frac{\alpha}{\theta \left( \alpha (1 + \zeta (1 - \eta_M)) + (1 - \alpha)(1 + \zeta \theta) \right)} \]  
(13.52 i)

From (13.51), unanticipated inflation is determined by,

\[ \pi_t - E_{t-\tau} \pi_t = \psi_A \epsilon_t^A + \psi_C \epsilon_t^C + \psi_S \epsilon_t^S + \psi_M \epsilon_t^M \]  
(13.53)

All the relevant shocks, real and nominal, affect unanticipated inflation. Thus, all the relevant shocks affect output fluctuations as well. Substituting (13.53) in the output supply function we get that fluctuations of output around its “natural” level are given by,

\[ y_t = \bar{y}_t + \frac{1 - \alpha}{\alpha} \left( (1 + \psi_A) \epsilon_t^A + \psi_C \epsilon_t^C + \psi_S \epsilon_t^S + \psi_M \epsilon_t^M \right) \]  
(13.54)

Thus, innovations in productivity, consumption demand, the money supply and money demand, cause deviations of output from its “natural” level. The variance of output around its natural level is given by,

\[ \text{Var}(y_t - \bar{y}_t) = \left( \frac{1 - \alpha}{\alpha} \right)^2 (1 + \psi_A)^2 \sigma_A^2 + \psi_C^2 \sigma_C^2 + \psi_S^2 \sigma_S^2 + \psi_M^2 \sigma_M^2 \]  
(13.55)
Thus, under an exogenous money supply rule, all shocks affect the variance of output around its “natural” level, as the money supply does not adjust to counteract the effects on these shocks on unanticipated inflation.

13.5 Aggregate Fluctuations under a Feedback Nominal Interest Rate Rule

In reality, modern central banks do not allow the money supply to follow an exogenous process of the form of (13.45). Monetary policy usually reacts to deviations of inflation from target, or to deviations of output and unemployment from target. In addition, central banks usually conduct monetary policy by controlling the nominal interest rate rather than the money supply. This is because of the difficulties in controlling the money supply, and because the money demand function is subject to shocks due to financial innovations.\(^7\)

In what follows we shall thus examine the behavior of the model under the assumption that the central bank follows a feedback rule for the nominal interest rate. In particular, we shall assume that the central bank follows a feedback rule of the form,

\[
    i_t = r_t + \mu + \phi_1 (\pi_t - \mu) + \phi_2 (y_t - y_\mu) + \epsilon_t
\]  

(13.56)

where \(\phi_1, \phi_2 > 0\) are policy parameters, and \(\epsilon_t\) is a white noise policy shock (error).

This is a generalization of the Wicksell rule that we examined in the case of the “new classical” model of Chapter 11. The generalization is due to Taylor (1993, 1999) who showed that in the last 35 years or so, the Federal Reserve, and other central banks, follow such feedback interest rate rules.

According to this rule, the central bank aims for a nominal interest rate which is equal to the “natural” real rate of interest, plus a target inflation rate equal to \(\mu\). If actual inflation is higher than the target \(\mu\), then the central bank raises interest rates in order to reduce inflation. In addition, if output is higher than its “natural” level, and unemployment lower than its “natural rate”, then the central bank also raises interest rates, in order to bring output back to its “natural” level and unemployment back to its “natural rate”.

Under this assumption, our “new” keynesian model thus consists of the output supply function (13.42), the Fisher equation (13.32), the real interest rate equation (13.44), and the policy rule (13.51). These equations can help determine the price level and inflation, output and the real interest rate. Once we determine inflation, we can also determine unanticipated inflation, and the evolution of employment, unemployment and real wages, through equations (13.40), (13.41) and (13.43).

Substituting the policy rule (13.56) in the Fisher equation (13.32), and using the real interest rate equation (13.44) and the output supply function (13.42), we get the following process for inflation.

\[
    \pi_t = \gamma_1 E_{t-1,1} \pi_t + \gamma_2 E_{t+1} \pi_t + (\phi_i - 1) \gamma_2 \mu - \gamma_1 \epsilon_{t-1} - \gamma_2 \epsilon_t
\]  

(13.57)

---

\(^7\) See Bernanke (2006) for how the Federal Reserve has been conducting monetary policy.
where, \( \gamma_1 = \frac{(\phi_2 + \theta)(1 - \alpha)}{\phi_1 \alpha + (\phi_2 + \theta)(1 - \alpha)} < 1 \), and, \( \gamma_2 = \frac{\alpha}{\phi_1 \alpha + (\phi_2 + \theta)(1 - \alpha)} \).

The inflationary process depends on the policy parameters of the Taylor rule and the other structural parameters of the model, such as \( \alpha \) and \( \theta \). It is driven by two shocks. Shocks to productivity, as these shocks cause deviations of output from its “natural” level, due to the fact that nominal wages were determined before the realization of these shocks, and also shocks to the policy rule (13.51). No other shocks affect the inflationary process under this rule, as the nominal interest rate adjusts to reflect changes in the “natural” rate of interest, which is affected by the other shocks.

The inflation process (13.57) is stable if \( \gamma_1 + \gamma_2 < 1 \). This requires that,

\[
\phi_1 > 1 \tag{13.58}
\]

Condition (13.58), is usually referred to as the Taylor principle, and requires that the nominal interest rate reacts more than one to one to deviations of current inflation from its target \( \mu \). This is a sufficient condition for a stable and determinate inflation process, and we shall assume that it is satisfied by the central bank.

If (13.58) is satisfied, then the rational expectations solution of the inflation process takes the form,

\[
\pi_t = \mu - \gamma_1 e^A_t - \gamma_2 e^i_t \tag{13.59}
\]

Inflation deviates from the central bank target \( \mu \), only in response to current shocks to productivity and shocks to the setting of the nominal interest rate.

From (13.59) unanticipated inflation is thus given by,

\[
\pi_t - E_{t-1} \pi_t = -\gamma_1 e^A_t - \gamma_2 e^i_t \tag{13.60}
\]

Substituting (13.60) in the output supply function (13.42), we get that,

\[
y_t = y_\gamma + \frac{1 - \alpha}{\alpha} \left( (1 - \gamma_1) e^A_t - \gamma_2 e^i_t \right) \tag{13.61}
\]

Fluctuations of output around its “natural” level depend positively on unanticipated shocks to productivity and negatively on nominal interest rate shocks. The impact of the shocks depends on the policy parameters of the central bank rule, which affect \( \gamma_1 \) and \( \gamma_2 \).

From (13.61) the variance of deviations of output from its “natural” level is given by,

\[
Var(y_t - y_\gamma) = \left( \frac{1 - \alpha}{\alpha} \right)^2 \left( (1 - \gamma_1)^2 \sigma_A^2 + \gamma_2^2 \sigma_i^2 \right) \tag{13.62}
\]

Substituting (13.60) in the employment equation (13.40), the unemployment equation (13.41), the real wage equation (13.43) and the real interest rate equation (13.44), we see that unanticipated
shocks to productivity and the nominal interest rate, also affect deviations of these variables from their “natural” levels as well.

From (13.41), the deviation of unemployment from its “natural rate” is determined by,

\[ u_t - u_t = -\frac{1}{\alpha} \left( \pi_t - E_{t-1} \pi_t + \epsilon_t^\alpha \right) \]  

(13.41')

Since unanticipated inflation and unanticipated productivity shocks are not persistent, deviations of unemployment from its “natural rate” will be non persistent either. For example, under a Taylor (1993) monetary policy rule, deviations of unemployment from its “natural rate” are determined by,

\[ u_t - u_t = \frac{1}{\alpha} \left( -(1 - \gamma_1) \epsilon_t^A + \gamma_2 \epsilon_t^i \right) \]  

(13.63)

From (13.61) and (13.63) we can confirm that under a Taylor rule, only productivity and monetary policy shocks affect fluctuations in real variables, such as output and unemployment, around their “natural” level. This is in contrast to the exogenous rule for monetary growth, which results in all shocks affecting deviations of output from its “natural” rate, and thus a higher potential variance of output.

Furthermore, the impact of these shocks in (13.61) and (13.62) depends on the parameters of the monetary policy rule (13.51), which can affect \( \gamma_1 \) and \( \gamma_2 \). Thus, in this model there is scope for monetary policy to affect the short run fluctuations of real variables by appropriate choice of the policy parameters.

We shall return to the issue of the appropriate role of monetary policy in Chapter 16.

A final remark is in order though. We can see from (13.59), (13.61) and (13.63) that fluctuations of inflation from target, and output and unemployment around their “natural” levels and are the sum of two white noise processes, i.e white noise processes themselves. All deviations last for one period and there is no persistence. This lack of persistence is a serious weakness of the model, as persistence of aggregate fluctuations is one of the main characteristics of business cycles.

### 13.6 Unemployment Persistence, Inflation and Monetary Policy

As we have presented it so far, this model can account for fluctuations of inflation around the target of the monetary authorities and output, employment and unemployment around their “natural” rates, but these fluctuations are not persistent.

Yet, persistence of aggregate fluctuations is one of the main characteristics of business cycles. To account for persistence in this model, the model must be generalized to introduce a propagation mechanism for the effects of the various shocks.

#### 13.6.1 Nominal Wage Contracts in a Dynamic Insider Outsider Model
One way to introduce unemployment persistence in a model with periodic wage setting has been proposed by Blanchard and Summers (1986). We shall examine a fully dynamic version of their model.\(^8\)

Following Blanchard and Summers (1986), we assume that the employment objective which determines the nominal wage in the contract depends on both the exogenous number of “core insiders” in each firm, but also those who were employed in period \(t-1\). The expectations on the basis of which wages are set depend on information available until the end of period \(t-1\), but not on information about prices and productivity in period \(t\).

On the basis of the above, we assume that the objective of “insiders” is to make expected employment satisfy a path that minimizes the following quadratic inter-temporal loss function,

\[
\min E_{t+1} \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{2} (l(i)_{t+1} - \bar{n}(i))^2 + \frac{\omega}{2} (l(i)_{t+1} - l(i)_{t+1-1})^2 \right] \tag{13.64}
\]

(13.64) is minimized subject to the sequence of labor demand equations (13.4), as employment in each period is determined ex post by the firm.

\(\bar{n}\) is the logarithm of the number of core “insiders”. \(\beta=1/(1+\rho)<1\) is the discount factor, with \(\rho\) being the pure rate of time preference. \(\omega\) is the weight of recent employees relative to “core insiders” in the wage setting process. As can be seen from (13.64), “outsiders”, i.e the unemployed of the previous period, have no influence on the wage setting process.\(^9\)

We shall assume that the total number of core “insiders” in the economy is always strictly smaller than the labor force. We shall thus assume that,

\[
\int_{i=0}^{1} n(i) di = \bar{n} < n, \forall t \tag{13.65}
\]

where \(n\) is the log of the labor force.

From the first order conditions for a minimum of (13.64), wages are set so that expected employment for each firm satisfies,

\[
(1 + \omega(1 + \beta)) E_{t-1} l(i) - \beta \omega E_{t-1} l(i)_{t+1} - \omega l(i)_{t-1} = \bar{n}(i) \tag{13.66}
\]

Integrating over the number of firms \(i\), expected aggregate employment must then satisfy,

\[
(1 + \omega(1 + \beta)) E_{t-1} l - \beta \omega E_{t-1} l_{t+1} - \omega l_{t-1} = \bar{n} \tag{13.67}
\]

---

\(^8\) The model we analyze in this section is due to Alogoskoufis (2015).

\(^9\) Note that (13.5) is a special case of (13.64) with \(\omega=0\). Thus, in (13.5) only core “insiders” and not those recently employed affect the wage setting process. An alternative interpretation of (13.64) would be in terms of “adjustment costs”. “Insiders” seek to minimize deviations of expected employment from their number, but there is a cost to adjusting employment from period to period. Then, \(\omega\) can be interpreted as the relative importance of adjustment costs relative to costs of deviations from the employment of core “insiders.”
(13.67) is the same as (13.66) without the $i$ index and refers to aggregate employment.

(13.67) helps explain the differences of our dynamic wage setting model from the model in the previous section, where only “core” insiders, and not the employees of the previous period affected wage contracts. In the model where only “core” insiders affect wage contracts, $\omega=0$, as recent employees do not exert any separate influence in the wage setting process. Setting $\omega=0$ in (13.67), nominal wages would be set in order to ensure that,

$$E_{t-1}l_t = \bar{n}$$

which is the same as equation (13.8). Thus, the dynamic model we are considering now, contains the static model of the previous sections as a special case.

In our more general forward looking dynamic model, from (13.67), expected employment is given by,

$$E_{t-1}l_t = \frac{1}{1+\omega(1+\beta)}\bar{n} + \frac{\omega}{1+\omega(1+\beta)}l_{t-1} + \frac{\beta\omega}{1+\omega(1+\beta)}E_{t-1}l_{t+1}$$

(13.68)

Thus, in our dynamic wage setting model, “insiders” set nominal wages in order to achieve an employment target which depends on core “insiders”, those previously employed, but also on expected future employment, as expected future employment will affect future wage setting behavior.

### 13.6.2 Wage Determination, Unemployment Persistence and the Phillips Curve

Subtracting (13.67) from the log of the labor force $n$, after some rearrangement, we get,

$$(1+\omega(1+\beta))E_{t-1}u_t - \beta\omega E_{t-1}u_{t+1} - \omega u_{t-1} = \bar{u}$$

(13.69)

where, $u_t = n_t - l_t$ is the unemployment rate, and $\bar{u} = n - \bar{n} > 0$ is the “natural” unemployment rate. The “natural rate” of unemployment in this model is defined in terms of the difference between the labor force and the number of core “insiders”. This is the equilibrium rate towards which the economy would converge in the absence of shocks.

To solve (13.69) for expected unemployment, define the operator $F$ as,

$$F^s u_t = E_{t-1}u_{t+s}$$

(13.70)

We can then rewrite (13.69) as,

$$\left((1+\omega(1+\beta))F^0 - \beta\omega F - \omega F^{-1}\right)u_t = \bar{u}$$

(13.71)

(13.71) can be rearranged as,
It is straightforward to show that if $0 < \beta < 1$ and $\omega > 0$ and finite, the characteristic equation of the quadratic in the forward shift operator (in brackets) has two distinct real roots, which lie on either side of unity. The two roots satisfy,

$$\lambda_1 + \lambda_2 = \frac{1 + \omega(1 + \beta)}{\beta \omega} \quad \text{and} \quad \lambda_1 \lambda_2 = \frac{1}{\beta}$$

(13.73)

Using (13.73) we can rewrite (13.72), as,

$$(F - \lambda_1)(F - \lambda_2)u_t = -\frac{1}{\beta \omega} u$$

(13.74)

Assuming $\lambda_1$ is the smaller root, we can solve (13.74) as,

$$E_{t-1}u_t = \lambda_1 u_{t-1} + \frac{\lambda_1}{\omega(1 - \beta \lambda_1)} u_t = \lambda_1 u_{t-1} + (1 - \lambda_1) \tilde{u}$$

(13.75)

(13.75), which is the rational expectations solution of (13.69), determines the path of expected unemployment implied by the wage setting behavior of “insiders”.

From (13.73), it is straightforward to show that an increase in $\omega$, the relative weight of recent employees in the wage setting process, results in an increase in $\lambda_1$, the coefficient that determines the persistence of expected unemployment. From (13.73), which defines the two roots, it follows that,

$$\frac{\partial \lambda_1}{\partial \omega} = \left( \frac{\lambda_1}{\omega} \right)^2 > 0$$

Thus, the higher the weight of recent employees relative to core “insiders” in the wage setting process, the higher the persistence of unemployment.\(^\text{10}\)

Actual unemployment, is determined from the employment decisions of firms, after information about prices, productivity and other shocks has been revealed.

Integrating the labour demand function (13.4) over the number of firms $i$, aggregate employment is given by,

$$l_t = l - \frac{1}{\alpha}(w_t - p_t - a_t)$$

(13.76)

\(^\text{10}\) For example, assuming $\beta=0.99$, with $\omega=1$, $\lambda_1=0.38$. With $\omega=2$, $\lambda_1=0.50$, with $\omega=10$, $\lambda_1=0.73$ and with $\omega=100$, $\lambda_1=0.91$.\)
Subtracting the aggregate employment equation (13.76) from the log of the labor force $n$, actual unemployment is determined by,

$$u_t = n - \bar{l} + \frac{1}{\alpha} (w_t - p_t - a_t)$$  \hspace{1cm} (13.77)

Taking expectations on the basis of information available at the end of period $t-1$, the wage is set in order to make expected unemployment equal to the expression in (13.75), which defines the rate of unemployment consistent with the wage setting behavior of “insiders”.

From (13.77), the wage is thus set in order to satisfy,

$$w_t = E_{t-1} p_t + E_{t-1} a_t + \alpha \left( E_{t-1} u_t - n + \bar{l} \right)$$  \hspace{1cm} (13.78)

where $E_{t-1} u_t$ is determined by (13.75).

Substituting for the nominal wage in (13.77), using (13.78), then the unemployment rate evolves according to,

$$u_t = E_{t-1} u_t - \frac{1}{\alpha} (p_t - E_{t-1} p_t + a_t - E_{t-1} a_t)$$  \hspace{1cm} (13.79)

Substituting (13.75) in (13.79) thus gives us the solution for the unemployment rate.

$$u_t = \lambda u_{t-1} + (1 - \lambda) u - \frac{1}{\alpha} (p_t - E_{t-1} p_t + a_t - E_{t-1} a_t)$$  \hspace{1cm} (13.80)

From (13.80), the unemployment rate is equal to the expected unemployment rate, as determined by the behavior of “insiders” in the labor market, and depends negatively on unanticipated shocks to inflation and productivity. Unanticipated shocks to inflation reduce unemployment by a factor which depends on the elasticity of labor demand with respect to the real wage, as unanticipated inflation reduces real wages. Unanticipated shocks to productivity also reduce unemployment, as they reduce the difference between real wages and productivity and increase labor demand.

We can express (13.80) in terms of inflation, by adding and subtracting the lagged log of the price level in the last parenthesis. Thus, (13.80) takes the form of a dynamic, expectations augmented “Phillips Curve”.

$$u_t = \lambda u_{t-1} + (1 - \lambda) u - \frac{1}{\alpha} (\pi_t - E_{t-1} \pi_t + a_t - E_{t-1} a_t)$$  \hspace{1cm} (13.81)

where $\pi$ is the inflation rate.

(13.81) can be expressed in terms of deviations of unemployment from its “natural” rate, as,
From (13.82), deviations of unemployment from its “natural” level depend negatively on unanticipated shocks to inflation and productivity, as these cause a discrepancy between real wages and productivity, due to the fact that nominal wages are predetermined. Unanticipated shocks to inflation reduce real wages and induce firms to increase labor demand and employment beyond their “natural” level. Thus, unemployment falls relative to its “natural” rate. Unanticipated shocks to productivity, given inflation, cause an increase in productivity relative to real wages, and also cause firms to increase labor demand, employment and output, beyond their “natural” levels, which reduces unemployment beyond its “natural” rate.

It can easily be confirmed from (13.82) that following a shock to inflation or productivity, unemployment will converge gradually back to its “natural” rate, with the speed of adjustment being \((1 - \lambda_1)\) per period. Thus, following shocks to inflation or productivity, deviations of unemployment from its “natural” rate will display persistence.

**13.6.3 The Relation between Output and Unemployment Persistence**

The persistence of employment and unemployment, will also be translated into persistent output fluctuations.

Aggregating the firm production functions (3), the aggregate production function can be written as,

\[
y_t = a_t + (1 - \alpha)\bar{y}_t
\]  

(13.83)

Adding and subtracting \((1 - \alpha)(n - \bar{n})\), the production function can be written as,

\[
y_t = \bar{y}_t - (1 - \alpha)(u_t - \bar{u})
\]  

(13.84)

where,

\[
\bar{y}_t = (1 - \alpha)n + a_t
\]  

(13.85)

is the log of the “natural” level of output.

(13.84) is an Okun (1962) type of relation, which suggests that fluctuations of output around its “natural” level will be negatively related to fluctuations of the unemployment rate around its own “natural” rate.

From (13.84) and (13.82), deviations of output from its “natural” level will be determined by,

\[
y_t - \bar{y}_t = \lambda_1(y_{t-1} - \bar{y}_{t-1}) + \frac{1 - \alpha}{\alpha}(\pi_t - E_{t-1}\pi_t + a_t - E_{t-1}a_t)
\]  

(13.86)
(13.86) shows that deviations of output from its “natural” level, also display persistence, because of the persistence of employment and unemployment.

(13.86) is a dynamic output supply function. Deviations of output from its “natural” level depend positively on unanticipated shocks to inflation and productivity, as these cause a discrepancy between real wages and productivity, due to the fact that nominal wages are predetermined. Unanticipated shocks to inflation reduce real wages and induce firms to increase labor demand, employment and output. Unanticipated shocks to productivity, given inflation, cause an increase in productivity relative to real wages, and also cause firms to increase labor demand, employment and output, beyond their “natural” levels. On the other hand, anticipated shocks to productivity increase both output and its “natural” level by the same proportion.

Under unemployment persistence, current employment, unemployment, output, real wages and the real interest rate, as functions of the exogenous shocks and shocks to inflation, evolve according to,

\[ l_t = (1 - \lambda_1)\bar n + \lambda_t l_{t-1} + \frac{1}{\alpha} \left( \pi_t - E_{t-1}\pi_t + \varepsilon_t^d \right) \]  \hspace{1cm} (13.87)

where \( \bar n \) is the “natural” level of employment.

\[ u_t = (1 - \lambda_1)\bar u + \lambda_t u_{t-1} - \frac{1}{\alpha} \left( \pi_t - E_{t-1}\pi_t + \varepsilon_t^d \right) \]  \hspace{1cm} (13.88)

where \( \bar u \) is the “natural” rate of unemployment.

\[ y_t = \bar y_t + \lambda_t (y_{t-1} - \bar y_{t-1}) + \frac{1 - \alpha}{\alpha} \left( \pi_t - E_{t-1}\pi_t + \varepsilon_t^d \right) \]  \hspace{1cm} (13.89)

where, \( \bar y_t = (1 - \alpha)\bar n + a_t \).

\[ w_t - p_t = (w - p)_t + \alpha \lambda_t (u_{t-1} - \bar u) - \left( \pi_t - E_{t-1}\pi_t + \varepsilon_t^d \right) \]  \hspace{1cm} (13.90)

where, \( (w - p)_t = a_t - \alpha(n - \bar l) \).

\[ r_t = \bar r_t + \theta (1 - \alpha)(1 - \lambda_t)(u_t - \bar u) \]  \hspace{1cm} (13.91)

where \( \bar r_t = \rho - \theta (1 - \eta_\lambda) a_t + (1 - \eta_\xi)\nu_t^C \).

The “natural” rates (or levels) of real variables evolve as functions of the exogenous real shocks. However, unanticipated inflation, and innovations in productivity, by reducing real wages relative to their “natural” level, cause persistent increases in employment and output above their “natural” level, and persistent reductions in unemployment, real wages and the real interest rate, below their “natural” rates.

**13.6.4 Fluctuations of Unemployment and Inflation under a Taylor Rule**

Assume that deviations in the unemployment rate persist as in (13.82), and that the central bank follows a Taylor rule of the form,
where \( \phi_1, \phi_2 > 0 \) and \( \varepsilon_i^j \) is a white noise interest rate policy shock. We have now expressed the Taylor rule in terms of deviations of unemployment and not output from its “natural” rate. This does not affect the results, as through the Okun type relation (13.84), deviations of unemployment from its “natural rate” are a linear function of deviations of output from its “natural level”.

Substituting (13.92) in the Fisher equation, after using the real interest rate equation (13.91) and the dynamic Phillips curve (13.88), we get the following process for inflation,

\[
\pi_t = \gamma_1 E_t \pi_{t+1} + \gamma_2 E_t \pi_t + \gamma_3 \pi_{t-1} + \gamma_4 \mu + \gamma_5 \varepsilon^i_t + \gamma_6 \varepsilon^i_{t-1} \tag{13.93}
\]

where,

\[
\begin{align*}
\gamma_1 &= \frac{\alpha}{\phi_1 \alpha + \phi_2 + \theta (1 - \lambda_1) (1 - \alpha) + \lambda_1 \alpha} \\
\gamma_2 &= \frac{\phi_2 + \theta (1 - \lambda_1) (1 - \alpha)}{\phi_1 \alpha + \phi_2 + \theta (1 - \lambda_1) (1 - \alpha) + \lambda_1 \alpha} \\
\gamma_3 &= \frac{\lambda_1 \phi_1 \alpha}{\phi_1 \alpha + \phi_2 + \theta (1 - \lambda_1) (1 - \alpha) + \lambda_1 \alpha} \\
\gamma_4 &= \frac{(\phi_1 - 1) (1 - \lambda_1) \alpha}{\phi_1 \alpha + \phi_2 + \theta (1 - \lambda_1) (1 - \alpha) + \lambda_1 \alpha} \\
\gamma_5 &= -\gamma_2 \\
\gamma_6 &= -\gamma_1 \\
\gamma_7 &= \lambda_1 \gamma_1
\end{align*}
\]

Note that, because of the persistence of unemployment, the inflationary process also displays persistence. It also depends on the current expectations about future inflation, through the definition of the real interest rate and on both parameters of the Taylor rule, as unanticipated inflation causes the unemployment rate and the real interest rate to deviate from their “natural rates”. Finally, because of the persistence in unemployment both current and past nominal interest rate shocks affect the inflationary process. The effects of productivity and nominal interest rate shocks on inflation also depend on the parameters of the Taylor rule.\(^{11}\)

In order to solve for inflation, we first take expectations of (13.93) conditional on information available up to the end of period \( t-1 \). This yields,

\[
E_{t-1} \pi_t = \frac{1}{\phi_1 + \lambda_1} E_{t-1} \pi_{t+1} + \frac{\phi_1 \lambda_1}{\phi_1 + \lambda_1} \pi_{t-1} + \frac{(\phi_1 - 1) (1 - \lambda_1)}{\phi_1 + \lambda_1} \mu + \frac{\lambda_1}{\phi_1 + \lambda_1} \varepsilon^i_{t-1} \tag{13.94}
\]

\(^{11}\) (13.93) being the inflationary process from a dynamic stochastic general equilibrium model, in which the policy rule of the monetary authorities is taken into account when agents form their expectations, it does not suffer from the Lucas (1976) critique. Changing the parameters of the policy rule, would also change the parameters of the inflationary process.
The process (13.94) has two roots, $\lambda_1$ and $\phi_1$, and will be stable if the two roots lie on either side of unity. Since $\lambda_1 < 1$, the expected inflation process will be stable if,

$$\phi_1 > 1$$ \hspace{1cm} (13.95)

Condition (13.95), is the *Taylor principle*. It requires that nominal interest rates over-react to deviations of current inflation from target inflation, in order to affect expected real rates. This is a sufficient condition for a stable and determinate process for expected (and actual) inflation.\(^{12}\)

If (13.95) is satisfied, then the solution for the expected inflation process (13.94) is given by,

$$E_t \pi_t = (1 - \lambda_1) \mu + \lambda_1 \pi_{t-1} + \frac{\lambda_1}{\phi_1} \varepsilon_{t-1}$$ \hspace{1cm} (13.96)

From (13.96), it follows that,

$$E_t \pi_{t+1} = (1 - \lambda_1) \mu + \lambda_1 \pi_t + \frac{\lambda_1}{\phi_1} \varepsilon_t$$ \hspace{1cm} (13.97)

Substituting (13.96) and (13.97) in the inflation process (13.93), the rational expectations solution for inflation is given by,

$$\pi_t = (1 - \lambda_1) \mu + \lambda_1 \pi_{t-1} - \psi_1 \varepsilon_t + \psi_2 \varepsilon_{t-1} + \psi_3 \varepsilon_{t-2}$$ \hspace{1cm} (13.98)

where,

$$\psi_1 = \frac{\phi_3}{\phi_1 \alpha + \phi_2 + \theta(1 - \lambda_1)(1 - \alpha)} < 1$$

$$\psi_2 = \frac{\phi_1 - \lambda_1}{\phi_1 \alpha + \phi_2 + \theta(1 - \lambda_1)(1 - \alpha)} > 0$$

$$\psi_3 = \frac{\lambda_1}{\phi_1} > 0$$

From (13.98), the fluctuations of inflation around the target of the monetary authorities $\mu$ are persistent, and depend on the current innovation in productivity and current and past interest rate shocks. Furthermore, the persistence of inflation is equal to the persistence of deviations of unemployment and other real variables, such as output, from their “natural” level.

The fluctuations of unemployment and output around their “natural” level are driven by unanticipated inflation and innovations in productivity. From (13.98), unanticipated inflation is determined by,

\(^{12}\) Woodford (2003), among others, contains a detailed discussion of the Taylor principle, and its significance for the resolution of the price level and inflation indeterminacy problem highlighted by Sargent and Wallace (1975) for non contingent interest rate rules.
Substituting (13.99) in the “dynamic” Phillips curve (13.82) and the dynamic output supply function (13.86), deviations of unemployment and output from their “natural” rates are determined by,

\[ \pi_t - E_{t-1} \pi_t = -\psi_1 \epsilon_i^\lambda - \psi_2 \epsilon_i^i \]  

(13.99)

Thus, under the Taylor rule (13.92), only innovations in productivity and nominal interest rate shocks induce fluctuations of deviations of unemployment and output from their “natural” rates. Other demand shocks, such as shocks to consumption preferences, are fully neutralized by monetary policy, since the nominal interest rate is assumed to fully accommodate changes in the “natural” rate of interest.

However, because of the persistence in deviations of unemployment and output from their “natural” levels, the effects of these shocks are no longer short lived, but they display persistence. The higher the persistence of deviations of unemployment from its “natural” rate, the higher the persistence of the effects of temporary nominal and real shocks.

13.6.5 A Dynamic Simulation of the Effects of Monetary and Real Shocks

In order to calculate the impulse response functions of the model to nominal and real shocks, we present the results of a dynamic simulation of the model, following an unanticipated temporary 1% shock to the nominal interest rate, and an unanticipated 1% shock to productivity respectively.

In the simulations we have assumed the following values of the parameters: \( \alpha=0.333, \rho=0.02, \theta=1, \omega=2 \) implying a value of \( \lambda_1=0.5, \phi_1=1.5, \phi_2=0.5 \) and \( \eta_A=0.75 \). We have also assumed a “natural rate” of unemployment equal to 5% and a target inflation rate of 2%.

In Figure 13.1 we present the impulse response functions of the model, following an unanticipated temporary 1% shock to the nominal interest rate. Inflation initially falls below the target of 2%, unemployment rises above its “natural” rate, and output falls below its own “natural” level. The real interest rate and the real wage rise above their “natural” levels. Because of the fall in inflation and the rise in unemployment, after the initial shock, the nominal interest rate follows a downward path towards its “natural rate”, inflation rises, unemployment gradually declines towards its “natural rate”, and all other real variables adjust towards their “natural levels”. Thus, a temporary nominal shock has persistent real effects, because of the persistence of deviations of employment from its “natural level”.

In Figure 13.2 we present the impulse response functions of the model, following an unanticipated 1% shock to productivity. Inflation initially falls, and so does unemployment and nominal and real interest rates relative to their “natural rates”. Output rises above its “natural” level, and so do real wages. Following the initial shock, all variables gradually return to their “natural levels”. Thus,
both nominal and real shocks cause persistent deviations of all variables from their steady state values.

13.7 Conclusions

In this chapter we have introduced a dynamic stochastic “new Keynesian” model, which not only allows for the existence of involuntary unemployment, but also for nominal shocks and monetary policy to affect the fluctuations of all real variables.

The model builds on one of the key insights of the General Theory, the short run rigidity of nominal wages, but in all other respects it is based on inter-temporal optimization on the part of both households and firms.

The model is characterized by an expectations augmented “Phillips curve”, in which deviations of output and employment from their “natural” level depend on unanticipated current inflation, which reduces real wages relative to productivity, and unanticipated productivity shocks, which also affect the relation between real wages and productivity.

Nominal shocks and, by extension, monetary policy are able to affect fluctuations in both inflation and real variables such as output, employment, unemployment, real wages and the real interest rate.

We first analyzed aggregate fluctuations in this model under two alternative monetary rules. The first is an exogenous process for the rate of growth of the money supply and the second is a feedback interest rate rule, according to which the nominal interest rate responds to deviations of inflation from the target of the central bank, and deviations of output from its “natural” level. Contrary to the “new classical” model, monetary shocks affect real variables in this model, causing temporary deviations of output, employment, unemployment, real wages and the real interest rate from their “natural” levels. The exact variance of such deviations depends on the monetary rule. Under an exogenous process for the rate of growth of the money supply, all shocks affect aggregate fluctuations. Under a Taylor feedback interest rate rule, only productivity shocks and shocks to monetary policy affect aggregate fluctuations. We have thus demonstrated the dependence of aggregate fluctuations not only on exogenous shocks, but on the form of the monetary policy rule followed by the central bank.

We have also extended the model to account for persistence in deviations of unemployment and output from their “natural” levels. The extension is based on a dynamic model of the “Phillips Curve”, in which unanticipated shocks to inflation and productivity have persistent effects on unemployment, and these persistent effects are compatible with full inter-temporal optimization on the part of labor market “insiders”. The propagation mechanism that causes unanticipated nominal and real shocks to produce persistent deviations of unemployment and output from their “natural” rate is the partial adjustment of labor market insiders to employment shocks. We demonstrate that under a Taylor rule, the only shocks that cannot be completely neutralized by monetary policy are productivity shocks and, of course, monetary policy shocks. Fluctuations of deviations of unemployment and output from their “natural” rates display persistence and are driven by these two types of shocks. Because of the endogenous persistence of deviations of unemployment from its “natural” rate, the equilibrium inflation rate also displays persistence around the inflation target of the central bank.
Figure 13.1
Impulse Response Functions following a 1% Unanticipated Temporary Shock to the Nominal Interest Rate
Figure 13.2
Impulse Response Functions following a 1% Unanticipated Shock to Productivity
References


