Chapter 14
A Model of Imperfect Competition and Staggered Pricing

In this chapter we present the structure of an alternative new Keynesian model of aggregate fluctuations. The model is a dynamic stochastic general equilibrium model based on monopolistic competition in product markets, and we analyze it assuming both full adjustment of wages and prices and staggered pricing. The model with full adjustment of wages and prices is comparable to the “new classical model” without capital, presented in Chapter 11, and the model with staggered pricing is comparable to the “new keynesian” model with periodic nominal wage contracts of Chapter 13.

The imperfectly competitive “new Keynesian” model has two important differences from the typical perfectly competitive “new classical” model.

First, instead of fully competitive markets for goods and services, it assumes that markets are characterized by conditions of monopolistic competition. Firms do not take prices as given, but determine prices so as to maximize profits. Because of monopolistic competition, employment, real output, consumption and real wages are determined at a lower level than in the corresponding competitive model, even when there is complete flexibility in prices and wages. However, by itself this difference does not result in major differences from the “new classical” competitive model regarding the nature of macroeconomic fluctuations.

Second, in the imperfectly competitive new Keynesian model it is usually assumed that firms adjust prices only gradually. Two observationally equivalent versions of gradual price adjustment have dominated the literature. The one is the Rotemberg (1982 a,b) model of monopolistic price adjustment, and the second is the Calvo (1983) model of staggered pricing. In the Rotemberg model, firms balance the costs of adjusting prices against the costs of deviating from the profit maximizing optimal price. They end up gradually adjusting prices, so as to gradually approach the optimal price. In the Calvo model, it is assumed that only a fixed proportion of firms have the freedom to adjust prices in any given period. This results in the remaining firms not being able to adjust prices. Although optimal pricing takes this restriction into account in advance, the aggregate price level adjusts only gradually. These two alternative assumptions lead to models with price level stickiness, which differ significantly from the new classical models, and share many of the properties of the “new keynesian” model with periodic nominal wage contracts analyzed in Chapter 13.

In the “new keynesian” model with periodic nominal wage contracts it is only deviations of current inflation from prior expectations of current inflation that result in deviations of output and unemployment from their “natural rates”. In the staggered pricing model, it is deviations of inflation from expected future inflation that are associated with such deviations. This results in a different type of Phillips curve, called the new keynesian Phillips curve, which differs from the traditional
expectations augmented Phillips curve, as current inflation depends on current expectations of future inflation, and not prior expectations of current inflation.

The imperfectly competitive new Keynesian model has the following structure:

Deviations of inflation from the target of the central bank are determined by the “new Keynesian” Phillips curve, and depend of expected future inflation and deviations of real output from its “natural” level, as the latter cause an increase in nominal marginal costs and hence prices.

The deviations of aggregate demand from the “natural” level of real output depend on the new Keynesian IS curve, which, as in the periodic nominal wage contracts model of Chapter 13, depend on deviations of the current real interest rate from its “natural” level.

The nominal interest rate is determined by the central bank, which follows a Taylor interest rate rule. According to the Taylor rule, the nominal interest rate reacts positively to deviations of current inflation from the central bank target, as well as deviations of real output from its “natural” level.

After presenting the properties of this model, we analyze the effects of monetary and real shocks on fluctuations in real output and the price level (inflation).

The imperfectly competitive new keynesian model with staggered prices can, unlike the classical model, explain monetary cycles, i.e aggregate fluctuations caused by monetary shocks. These shocks are transmitted to real variables, and, to the extent that they persist over time, have persistent real effects. However, due to the absence of labor market distortions, this model cannot account for “involuntary” unemployment. A more satisfactory “new keynesian” model must combine the labor market distortions of the “periodic nominal wage contracts” model of Chapter 13, with the product market distortions of the model of this chapter.

14.1 An Imperfectly Competitive New Keynesian Model

In this section we examine in detail the structure of an imperfectly competitive new Keynesian model. The basic model that we analyze has much in common with the “new” classical model. It is a dynamic stochastic general equilibrium model with two important differences from the “new” classical model.¹

First, instead of perfectly competitive markets for goods and services we assume that markets are characterized by conditions of imperfect (monopolistic) competition. Firms do not take prices as given, but have the power to determine prices that maximize profits. Because of imperfect competition, in equilibrium, employment, real output, consumption and real wages are determined at a lower level than in the corresponding competitive model, even when there is complete flexibility in prices and wages. However, by itself this difference does not result in material differences from the competitive classical model with respect to the nature of aggregate fluctuations. If this was the only difference, we could well talk about an imperfectly competitive “new classical” model.

¹ See Gali (2008) for a fuller presentation and analysis of this model.
Second, we assume that there is staggered price adjustment, i.e. that firms do not have the ability to change their prices at all times. This assumption is what makes the model “new keynesian”, as it leads to a model in which the price level adjusts gradually towards the equilibrium price level. As a result of gradual price adjustment, real variables deviate from their “natural” rates, and monetary shocks can have real effects, as was the case with the model with predetermined wages examined in Chapter 13.

Gradual adjustment of nominal wages and prices is thus the key element that differentiates the new keynesian from the new classical approach to macroeconomic fluctuations. With full and immediate adjustment of wages and prices, nominal disturbances do not affect real variables, even if there is imperfect competition.

14.1.1 The Representative Household

The problem of the representative household under monopolistic competition has one difference from the corresponding problem under perfect competition. The difference is that because of monopolistic competition, the household consumes differentiated products.

The representative household maximizes,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t u(C_t, N_t)$$

(14.1)

where $C$ is consumption, $N$ is labor supply, and $\rho$ is the pure rate of time preference. Consumption consists of all produced goods, which are defined on the basis of a constant index $j$ in the interval $[0,1]$. Aggregate consumption is thus given by,

$$C_t = \left( \int_{j=0}^{\varepsilon} C_t(j)^{\varepsilon-1} \ v_j \right)^{\varepsilon}$$

(14.2)

where $\varepsilon$ is also a parameter of the preferences of the representative household, and more precisely, the elasticity of substitution between goods. We assume that $\varepsilon > 1$.

The sequence of budget constraints under which the household maximizes inter-temporal utility is given by,

$$\int_{j=0}^{\varepsilon} P(j)C_t(j) v_j \leq B_t - B_{t-1} + W_t N_t - T_t$$

(14.3)

The household must also satisfy the transversality condition,

$$\lim_{T \to \infty} E_T B_T \geq 0$$

(14.4)

where $P(j)$ is the price of good $j$, $W$ the nominal wage, $i$ the nominal interest rate, $B$ a nominal one period bond, and $T$ an exogenous transfer of nominal income to the household (dividends, government transfers or taxes).
Apart from the decision about aggregate consumption and labor supply, which we have analyzed in the relevant section of Chapter 11, the household must now decide on the distribution of its consumption expenditure among the various goods. This requires the maximization of the consumption bundle (14.2) for any level of expenditure. One can easily deduce that this implies,

\[ C_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} C(t) \]  

(14.5)

for any good \( j \) in the interval \([0,1]\), where \( P \) is the average price level, defined as,

\[ P_t = \left( \int_{j=0}^{1} P_t(j)^{1-\varepsilon} \, dj \right)^{1/\varepsilon} \]  

(14.6)

In addition, when the household follows this optimal allocation policy, we also have that,

\[ \int_{j=0}^{1} P_t(j) C_t(j) \, dj = P_t C_t \]  

(14.7)

(14.7) suggests that total consumption expenditure can be written as the product of the aggregate consumption index and the aggregate price index. Substituting (14.7) in the sequence of budget constraints (14.3), we get,

\[ P_t C_t + \frac{1}{1+i_t} B_t \leq B_{t-1} + W_t N_t - T_t \]  

(14.8)

This sequence of budget constraints is the same as the sequence of budget constraints of the representative household in the competitive classical model.

As a result, the first order conditions for consumption and labor supply are analogous to the ones of the new classical model we analyzed in Chapter 11.

\[ -\frac{u_{N_t}}{u_{C_t}} = \frac{W_t}{P_t} \]  

(14.9)

\[ \frac{1}{1+i_t} = \frac{1}{1+\rho} E_t \left\{ \frac{u_{C_t}}{u_{C_t}} \frac{P_t}{P_{t+1}} \right\} \]  

(14.10)

We assume, as in the “new classical” model without capital of Chapter 11, that the utility function is defined by,

\[ U(C_t, N_t) = \frac{C_t^{1-\theta}}{1-\theta} - \frac{N_t^{\lambda+1}}{1+\lambda} \]  

(14.11)
where $1/\theta$ is the inter-temporal elasticity of substitution in consumption, and $1/\lambda$ the inter-temporal elasticity of substitution of labor supply.

Assuming that preferences take the form of (14.11), the first order conditions (14.9) and (14.10) can be written in log-linear form as,

$$w_t - p_t = \theta c_t + \lambda n_t \tag{14.12}$$

$$c_t = E_t(c_{t+1}) - \frac{1}{\theta}(i_t - E_t(\pi_{t+1}) - \rho) \tag{14.13}$$

where lower case letters denote the logarithms of the corresponding variables. $\pi$ is the rate of inflation. (14.12) and (14.13) are analogous to the ones in the “new” classical model without capital in Chapter 11.

### 14.1.2 The Representative Firm and Optimal Pricing

We assume that output is produced by a set of firms denoted by a continuous index $j$ defined in the interval $[0,1]$. Each firm produces a differentiated product under conditions of monopolistic competition. All firms have access to the same production technology, denoted by the production function,

$$Y_t(j) = A_t L_t(j)^{1-\alpha} \tag{14.14}$$

where $A > 0$ and $0 < \alpha < 1$ are exogenous technological parameters, common to all firms. $L(j)$ is employment of labor by firm $j$. The parameter $\alpha$ is constant, while $A$ is assumed to follow an exogenous stochastic process.

The optimal price of each firm, if it can choose its price in every period, is given by the maximization of its profits, under the constraint of the production function (14.14) and the demand function for its product (14.5). Each firm takes the average price $P$, the average wage $W$ and the level of total demand $C$ as given.

The per period profits of firm $j$ are given by,

$$P_t(j)Y_t(j) - W_tL_t(j) \tag{14.15}$$

From the first order conditions for a maximum of (14.15), under the constraints (14.14) and (14.5), the optimal price is determined as,

$$P_t(j) = \frac{W_t}{\varepsilon - 1} \left( \frac{W_t}{(1-\alpha)A_t L_t(j)^{-\alpha}} \right) \tag{14.16'}$$

The optimal price is a fixed multiple of the firm’s marginal cost, which equals the expression in brackets. The multiple depends on the elasticity of substitution between goods in the preferences of consumers, which determines the price elasticity of demand of their product, and therefore the profit margin of the firm. In the case of perfect competition that we examined in Chapter 11, the
elasticity of substitution tends to infinity, and the price tends to marginal cost. In the case of monopolistic competition with $\varepsilon > 1$, as we have assumed, the optimal price is higher than the marginal cost of labor.

As all firms have the same production function and face the same nominal wage and the same demand function for their product, they will all choose the same price. Consequently, the price level is defined as,

$$P_t = \frac{\varepsilon}{\varepsilon - 1} \left( \frac{W_t}{(1-\alpha)A_t L_t^{-\alpha}} \right) \quad (14.16)$$

Taking the logarithm of the production function (14.14) for the representative firm, and equation (14.16) for the optimal price, we get,

$$y_t = a_t + (1-\alpha)l_t \quad (14.17)$$

$$w_t - p_t = a_t - \alpha l_t - \mu \quad (14.18)$$

where,

$$a_t = \ln A_t, \quad \mu = \ln \left( \frac{\varepsilon}{\varepsilon - 1} \right) - \ln(1-\alpha).$$

$a$ is the logarithm of the exogenous productivity shock, and the constant $\mu$ is the logarithm of the markup on marginal cost, minus the logarithm of the coefficient of decreasing returns to labor.

14.1.3 Equilibrium with Full Price Flexibility

Solving the model under the assumption of full flexibility of prices, one can show that fluctuations in employment, output, consumption and real wages are a function only of the exogenous shocks to productivity, while fluctuations in the real interest rate are a function of the expected change in productivity; just as in the classical model with the assumption of perfect competition.

In the basic form of this model we shall assume that there is no investment or public consumption. Thus, in equilibrium, labor supply would be equal to labor demand by firms, and consumption will be equal to output.

$$n_t = l_t \quad (14.19)$$

$$y_t = c_t \quad (14.20)$$

The model consists of equations (14.12), (14.13), (14.17) and (14.18) and the equilibrium conditions (14.19) and (14.20). The model determines employment, output, consumption, real wages and the real interest rate as functions of the exogenous shock to productivity $a$.

The real interest rate is determined by the Fisher equation as,
Solving the model for the five endogenous variables, we get,

\[ l^N_t = n^N_t = \phi a_t + \tilde{n} \]  
\[ y^N_t = c^N_t = \psi a_t + \tilde{y} \]
\[ (w - p)^N_t = \chi a_t + \tilde{\omega} \]
\[ r^N_t = \rho + \theta \psi E_t (\Delta a_t) \]

where, \( \phi = \frac{1 - \theta}{\theta(1 - \alpha) + \alpha + \lambda} \) and, \( \tilde{n} = -\frac{\mu}{\theta(1 - \alpha) + \alpha + \lambda} \).

where, \( \psi = 1 + (1 - \alpha) \phi = \frac{1 + \lambda}{\theta(1 - \alpha) + \alpha + \lambda} \) and, \( \tilde{y} = (1 - \alpha) \tilde{n} \).

where, \( \chi = 1 - \alpha \phi = \frac{\theta + \lambda}{\theta(1 - \alpha) + \alpha + \lambda} \) and, \( \tilde{\omega} = (\theta(1 - \alpha) + \lambda) \tilde{n} \).

where, \( \psi = 1 + \frac{\theta + \lambda}{\theta(1 - \alpha) + \alpha + \lambda} \) and, \( \tilde{\omega} = (\theta(1 - \alpha) + \lambda) \tilde{n} \).

\[ r^N_t = \rho + \theta \psi E_t (\Delta a_t) \]

(14.22), (14.23), (14.24) and (14.25), along with the equilibrium conditions (14.19) and (14.20), determine the five endogenous real variables as functions of the exogenous productivity shock. Superscript \( N \) (natural) denotes the equilibrium values of the relevant variables, which, according to the Friedman definition are their “natural” rates.

Output, consumption and real wages are positive functions of the productivity shock \( a \), while employment is a positive function of the productivity shock only if \( \theta < 1 \), i.e. only if the elasticity of inter-temporal substitution is greater than one. If \( \theta > 1 \) employment is a negative function of productivity, while if \( \theta = 1 \) employment is independent of productivity. This applies because if \( \theta < 1 \) the inter-temporal substitution effect dominates on the income effect, following a change in productivity and real wages. If \( \theta > 1 \) the income effect dominates on the inter-temporal substitution effect, which in the case where \( \theta = 1 \) the two effects cancel each other out, and employment is not affected.

No other factor affects fluctuations in real variables. We see that, as in the competitive real business cycle model, monetary factors such as the money supply and nominal interest rates have no effect on the evolution of real variables.

However, in this model there is a significant difference from the competitive model of Chapter 11. Because of monopolistic competition, which implies a positive margin of prices over marginal costs of firms, both employment and output, as well as consumption and real wages, are determined at a lower level than in the case of perfect competition. Monopolistic competition implies a distortion
in the market of goods and services, which leads to lower equilibrium employment and output and to lower real wages than with perfect competition.\(^2\)

If the productivity shock follows a stationary stochastic process with mean zero, then, from (14.22), the log of the steady state employment level will be equal to,

\[
\frac{-\mu}{\theta(1-\alpha)+\alpha+\lambda} = \frac{\ln(1-\alpha) - \ln(\varepsilon / (\varepsilon - 1))}{\theta(1-\alpha)+\alpha+\lambda}
\]

If \(\varepsilon > 1\), the steady state employment level will be lower than in the case of perfect competition.

Under perfect competition, goods are perfect substitutes in the preferences of consumers. Thus, steady state employment would be equal to,

\[
\lim_{\varepsilon \to \infty} n = \frac{\ln(1-\alpha)}{\theta(1-\alpha)+\alpha+\lambda}
\]

Thus, because of imperfect competition, this model implies under employment relative to a fully competitive model, even when there is full flexibility of prices and wages. Through (14.23) and (14.24), this under employment implies that steady state output and steady state real wages will also be lower compared to perfect competition.

In all other respects, this model resembles the “new classical” competitive real business cycle model analyzed in Chapter 11.

14.1.4 Staggered Price Adjustment

In contrast to the “new classical” model, in “new keynesian” models one assumes gradual and not full adjustment of wages and prices towards their equilibrium values. In Chapter 13 we analyzed a model of predetermined nominal wages, which were set at the beginning of each period, and assumed fully flexible prices. Here we shall assume gradual adjustment of prices and fully flexible wages in a competitive labor market.

A number of alternative “new keynesian” models of gradual price adjustment under monopolistic competition have been developed in the literature. We shall concentrate on one of them, the Calvo (1983) model, which is based on staggered pricing.\(^3\)

Following Calvo (1983), we shall assume that firms cannot freely adjust their prices in every period. For each firm, the probability of adjusting prices in any period is equal to \(1-\gamma\), which is constant and independent of the length of time that has elapsed since the last price adjustment by the firm. Thus, in each period, a proportion \(1-\gamma\) of all firms adjust their prices, and the remaining proportion \(\gamma\) do not adjust their prices. As with predetermined nominal wage contracts in the model

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\(^2\) See Akerlof and Yellen (1985), Mankiw (1985), Blanchard and Kiyotaki (1987) and Ball and Romer (1990) for the first generation of “new keynesian” models that relied on monopolistic competition.

\(^3\) An observationally equivalent model, the Rotemberg (1982 a,b) model of quadratic costs of adjusting prices, is analyzed in the Annex to this Chapter.
of Chapter 13, this assumption has critical implications for the properties of the model, the nature of aggregate fluctuations and the effects of monetary shocks and monetary policy.\footnote{See Yun (1996) for the first analysis of the “new keynesian” dynamic stochastic general equilibrium model under the assumption that prices are set as postulated by Calvo (1983).}

Under this assumption, in period $t$, the expected future duration of any price contract is given by,

$$(1 - \gamma)\sum_{s=0}^{\infty} s\gamma^s = \frac{\gamma}{1 - \gamma}$$

From the definition of the price level, and the fact that all firms that reset their prices in period $t$ set the same price, it follows that,

$$P_t = \left( \gamma (P_{t-1})^{1-\varepsilon} + (1 - \gamma) (\bar{P}_t)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$  \hspace{1cm} (14.26)

where $\bar{P}$ is the price set by the firms that reset their prices in the current period.

From (14.26) one can show that the dynamic adjustment of the price level is given by,

$$\left( \frac{P_t}{P_{t-1}} \right)^{1-\varepsilon} = \gamma + (1 - \gamma) \left( \frac{\bar{P}_t}{P_{t-1}} \right)^{1-\varepsilon}$$  \hspace{1cm} (14.27)

In the steady state with zero inflation we have that,

$$P_t = P_{t-1} = \bar{P}_t$$  \hspace{1cm} (14.28)

A linear logarithmic approximation of (14.27) around the zero inflation steady state yields,

$$p_t - p_{t-1} = (1 - \gamma) \left( \bar{P}_t - P_{t-1} \right)$$  \hspace{1cm} (14.29)

From (14.29) it follows that inflation is positive if firms that set prices in the current period set them at a higher level than the average price of the previous period.

14.1.5 Optimal Pricing with Staggered Price Adjustment

In order to analyze the adjustment of inflation, one thus has to examine how firms decide on their optimal price, taking into account the fact that for a period in the future they may not be able to readjust their prices, while their competitors have the option of readjusting their own prices.

The problem of the firm that decides on the price it is about to set in period $t$ is to set the price that maximizes the expected present value of its profits, given that the probability of readjusting its price in any future period is equal to $1 - \gamma$. Thus, all firms that readjust their prices in period $t$ maximize,
under the constraints of the production function,

\[ L'_{t+s} = \left( \frac{Y'_{t+s}}{A_{t+s}} \right)^{\frac{1}{1-\alpha}} \]  (14.31a)

and the demand function,

\[ Y'_t = \left( \frac{P_t}{P_{t+s}} \right)^{\varepsilon} Y_{t+s} \]  (14.31b)

where, \( Y'_{t+s} \) and \( L'_{t+s} \) is the volume of output and employment in period \( t+s \), of the firm that has set its prices in period \( t \). The higher the relative price of the firm in any period, the lower the demand for its product and thus the volume of its output and employment.

From the first order conditions for a maximum it follows that,

\[
\sum_{s=0}^{\infty} \gamma^s E_t \left( \prod_{i=0}^{t} \left( \frac{1}{1+i_z} \right) \left( \frac{\bar{P}_t}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \right) - \frac{\varepsilon}{1-\alpha} \left( \frac{\bar{P}_t}{P_{t+s}} \right)^{\frac{1-\alpha+\varepsilon}{1-\alpha}} \left( \frac{W_{t+s}}{P_{t+s}} \right)^{\frac{1}{1-\alpha}} \left( \frac{Y_{t+s}}{A_{t+s}} \right)^{\frac{1}{1-\alpha}} = 0
\]  (14.32)

(14.32) implies that the expected present value of revenues from the optimal price is equal to the expected present value of the marginal cost of production, augmented by the profit margin \( \varepsilon/(\varepsilon-1) \) of the firm.

It is worth noting that, as we have already shown (equation (14.16)), if the firm could determine its prices in every period, the price of the product in each period would be equal to the marginal cost of production plus the same profit margin. However, if the firm cannot adjust prices in every period, as is assumed in the Calvo (1983) model, pricing follows the rule (14.32).

Assuming that in the steady state inflation is equal to zero, (14.32) can be transformed in logarithmic deviations from the steady state equilibrium, using a log linear Taylor approximation. Thus, in logarithms we shall have that,

\[
\bar{p}_t \approx (1-\beta \gamma) \sum_{s=0}^{\infty} \left( \beta \gamma \right)^s E_t \left( p_{t+s} + \omega \left( \mu + w_{t+s} - P_{t+s} + \frac{1}{1-\alpha} (\alpha y_{t+s} - a_{t+s}) \right) \right)
\]  (14.33)

where,
\[
\beta = \frac{1}{1 + \rho} \quad \text{and} \quad \omega = \frac{1 - \alpha}{1 - \alpha + \varepsilon} < 1.
\]

Consequently, firms that reset their prices in period \( t \) will choose a price which corresponds to a weighted average of the current and expected future price levels, plus a margin \( \mu \) on a weighted average of the current and expected future level of real marginal costs. The discount factor of a future period \( t+s \) depends on the probability that the firm will not be able to reset its price in the future period \( t+s \), which equals \( \gamma^s \), times the discount rate \( \beta^s \). Furthermore, the part of pricing which depends on the expected marginal cost of the firm depends negatively on the elasticity of demand for the product of the firm, through the parameter \( \omega \).

Using the future mathematical expectations operator \( F \), (14.33) can be written as,

\[
\frac{p_t - 1 - \beta \gamma}{1 - \beta \gamma F} p_t + \omega \left( \mu + \frac{1 - \beta \gamma}{1 - \beta \gamma F} \left( w_t - p_t + \frac{1}{1 - \alpha} (\alpha y_t - a_t) \right) \right)
\]

(14.34)

Substituting (14.34) in the equation for the adjustment of the average price level (14.29) we get that,

\[
p_t = \gamma p_{t+1} + (1 - \gamma) \left( \frac{1 - \beta \gamma}{1 - \beta \gamma F} p_t + \omega \left( \mu + \frac{1 - \beta \gamma}{1 - \beta \gamma F} \left( w_t - p_t + \frac{1}{1 - \alpha} (\alpha y_t - a_t) \right) \right) \right)
\]

(14.35)

Multiplying both sides of (1.35) by \( 1 - \beta \gamma F \), after some rearrangements, we get that,

\[
(1 + \beta) p_t - p_{t+1} - \beta E_t p_{t+1} = \frac{(1 - \gamma)(1 - \beta \gamma)}{\gamma} \omega \left( \mu + \left( w_t - p_t + \frac{1}{1 - \alpha} (\alpha y_t - a_t) \right) \right)
\]

(14.36)

(14.36) is the equation of adjustment of the price level towards the steady state price level, which is a constant markup on the marginal cost of production.

In order to examine the short run behavior of the model, we must introduce the equilibrium conditions in the markets for goods and services, labor and money.

14.1.6 Equilibrium in the Market for Goods and Services and the New Keynesian IS Curve

Equilibrium in the market for good \( j \) implies that,

\[
Y_t(j) = C_t(j)
\]

As a result, equilibrium in the market for all goods requires that,

\[
Y_t = C_t
\]

(14.37)

where \( Y \) is total output, defined in the same way as total consumption \( C \) in equation (14.2).
Substituting the Euler equation for consumption (14.13) in the equilibrium condition (14.37), the logarithm of real output is determined by,

\[
y_t = E_t(y_{t+1}) - \frac{1}{\theta}(i_t - E_t\pi_{t+1} - \rho)
\]

(14.38)

As in the “predetermined nominal wage contracts” model of Chapter 13, (14.38) is often referred to as the new keynesian IS curve, as it is derived from the equilibrium condition for the market for goods and services. Compared to the conventional IS curve, (14.38) contains the rational expectation about the future volume of output and depends on the real and not just the nominal interest rate. Its advantage over the conventional IS curve is that it has been derived from firm microeconomic foundations, and that its parameters depend on deep structural parameters, such as the pure rate of time preference of the representative household \(\rho\), and the inter-temporal elasticity of substitution in consumption \(1/\theta\).

14.1.7 Labor Market Equilibrium and the New Keynesian Phillips Curve

We next turn to the equilibrium condition in the labor market. We assume that in contrast to product prices that adjust gradually, nominal wages adjust immediately in order to equate the demand and supply of labor in each period. This assumption, is the exact opposite of the assumption made in the “periodic wage contracts” model of Chapter 13, and is made for reasons of analytical simplicity. Thus, the only stickiness which is analyzed in this version of the new Keynesian model is the gradual adjustment of prices rather than wages. This means that fluctuations in employment are the result of inter-temporal substitution by households and that no involuntary unemployment exists. Gali (2011) and others have analyzed this model with the additional assumption of rigidity not only in prices but also in nominal wages. In this case there are fluctuations in the unemployment rate due to the fact that wages are not equate the demand with the supply of labor in each period, as is assumed here.

Due to the gradual adjustment of prices, firms produce so as to satisfy aggregate demand at the given prices in each period. Aggregate output is determined at the level which is determined by aggregate demand, and differs from its “natural level”, which is the level that would prevail if there was immediate price adjustment by all firms.

As a result, aggregate output, employment, consumption, real wages and the real interest rate, differ from their “natural levels” and display fluctuations which depend on nominal as well as real disturbances.

From the price adjustment equation (14.36), we can deduce an equation for fluctuations in inflation. Expressing (14.36) as an inflation equation we have that,

\[
\pi_t = \beta E_t\pi_{t+1} + \frac{(1-\gamma)(1-\beta\gamma)}{\gamma} \omega \left( \mu + \left( \omega_t - p_t + \frac{1}{1-\alpha} \left( \alpha y_t - a_t \right) \right) \right)
\]

(14.39)

where \(\pi\) is the rate of inflation, defined as,

\[
\pi_t = p_t - p_{t-1}
\]
(14.39) implies that current inflation is greater than discounted future inflation, if the current marginal cost of labor, plus the margin $\mu$ is higher than the current price level $p$. The reason is that firms setting prices in the current period post larger price increases than (discounted) expected future inflation, in order to offset the higher current marginal cost of labor.

The assumption of equilibrium in the labor market means that we can substitute the real wage in (14.39) from the first order condition (14.12) for the representative household. Using (14.12), the condition for equilibrium in the market for goods and services $c=y$, and the production function (14.17), (14.39) can be rewritten as,

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - y_t^N)$$

(14.40)

where $y^N$ is the “natural rate” of real output, i.e. the output that would be produced if there was full flexibility of prices, and is given by (14.23). The parameter $\kappa$ is defined as,

$$\kappa = \frac{(1-\gamma)(1-\beta \gamma) \theta (1-\alpha) + \lambda + \alpha}{1-\alpha + \epsilon} > 0.$$

(14.40) is referred to as the new keynesian Phillips curve, and constitutes the second important behavioral equation of the imperfectly competitive new keynesian dynamic stochastic general equilibrium model.

The reason that deviations of output from its “natural rate” cause inflation to increase relative to expected future inflation is that higher output implies a higher real marginal cost of labor, and thus calls for price increases by firms that have the opportunity to change their current price which exceed discounted expected future inflation.

Like the new keynesian IS curve, the new keynesian Phillips curve has been derived from explicit microeconomic foundations, and its parameters are functions of deep structural parameters describing the preferences of households, the technology of production and the price setting technology.

14.1.8 The Structure of the New Keynesian Model with Staggered Pricing

Equations (14.38) and (14.40), along with equations (14.23) and (14.25) for the “natural level” of real output and the real interest rate constitute the basic structure of the imperfectly competitive “new keynesian” model.

Deviations of inflation from discounted expected future inflation are determined by the new keynesian Phillips curve (14.40), as a function of deviations of real aggregate demand and output from the “natural level” of output.

Deviations of real output from its “natural level” are determined by the new keynesian IS curve, which depends on deviations of the real interest rate from its “natural level”. The new keynesian IS curve can be expressed as,
where the natural levels of output and the real interest rate $y^N$, $r^N$ are determined by (14.23) and (14.25).

In order to close the model we must consider the determination of the nominal interest rate. In contrast to the “new classical” model, due to staggered price adjustment, fluctuations in real variables cannot be determined without reference to monetary factors. Monetary factors and monetary policy determine not only the price level and inflation, as in the “new classical” model, but also fluctuations in real variables such as real output, consumption and employment, real wages and the real interest rate.

14.1.9 The Taylor Rule for the Nominal Interest Rate

We shall analyze the model under the assumption that the central bank follows a Taylor (1993) rule of the form,

$$i_t = \rho + \eta_\pi \pi_t + \eta_y (y_t - y^N_t) + v_t$$  \hspace{1cm} (14.42)

where $\eta_\pi$ and $\eta_y$ are positive coefficients, and $v$ is an exogenous stochastic disturbance in the nominal interest rate. It is worth noting that because the constant in this rule is equal to $\rho$, this rule is consistent with zero steady state inflation.\(^5\)

As we already mentioned in Chapter 13, this rule implies a countercyclical monetary policy. When inflation is positive, the central bank increases nominal interest rates in order to reduce it. When employment is low, i.e. when output is lower than its “natural” level, the central bank reduces nominal interest rates in order to increase employment and nudge output towards its “natural” level. As we have shown already in Chapter 13, this feedback interest rate rule does not result in inflation and price level indeterminacy if the Taylor principle is satisfied, i.e. if the reaction of nominal interest rates to inflation is sufficiently strong.\(^6\)

Having now fully determined the “new keynesian” model with staggered pricing, we can analyze how nominal and real disturbances produce aggregate fluctuations.

14.2 Stochastic Shocks and Aggregate Fluctuations

Substituting (14.42) for the nominal interest rate in the “new keynesian” IS curve, and using the “new keynesian” Phillips curve, the model can be written in matrix form, as,

\[^5\] Note that the Taylor rule assumed is simpler than the Taylor rule we postulated in the model of Chapter 13. The nominal interest rate does to react to shocks that change the “natural rate” of interest, such as productivity shocks, and thus productivity shocks turn out to affect deviations of output from its “natural rate” and inflation. Obviously, one can also analyze the imperfectly competitive new keynesian model under the assumption that monetary policy follows a rule for the money supply and not nominal interest rates. See Gali (2008).

\[^6\] See Chapters 10, 11 and 13 for discussions of the properties of interest rate rules, as well as Woodford (2003), for a more extensive and complete analysis.
where, $\tilde{y}_t = y_t - y^N_t$ is the percentage deviation between current real output and its “natural” level.

The percentage deviation between current real output and its “natural” level is often referred to as excess output. Its opposite, is referred to as the output gap. When excess output is positive, the economy produces more than its “natural” level, while when it is negative (the output gap is positive) it produces less than its “natural” level.

We can see from (14.43) that the fluctuations of excess output and inflation depend on both types of shocks. Real shocks which affect $r^N - \rho$, and the nominal interest rate shock $v$.

The parameters determining aggregate fluctuations in the imperfectly competitive new keynesian model depend on the preferences of the representative household, ($\theta, \lambda, \varepsilon$ and $\rho$), the technology of production ($\alpha$), market structure ($\varepsilon$), the price adjustment mechanism ($\gamma$) and the parameters of the monetary policy rule ($\eta_\pi$ and $\eta_y$).

Given that both excess output and inflation are non predetermined variables, the solution will be unique only if the matrix of coefficients of future expectations has both eigenvalues inside the unit circle.\(^7\)

Under the assumption that the coefficients of the Taylor rule $\eta_\pi$ and $\eta_y$ are positive, one can show that a necessary and sufficient condition for a unique solution is,\(^8\)

$$\kappa(\eta_\pi - 1) + (1 - \beta)\eta_y > 0$$ (14.44)

In what follows we shall assume that (14.44) is satisfied. (14.44) requires a sufficiently pronounced reaction of nominal interest rates to inflation, as, solving for $\eta_\pi$, (14.44) can be expressed as,

$$\eta_\pi > 1 - \frac{(1 - \beta)}{\kappa} \eta_y$$

For example, if the reaction of the nominal interest rate to excess output is zero ($\eta_y = 0$), then the reaction of nominal interest rates to inflation must exceed unity.

14.2.1 Implications of a Nominal Interest Rate Shock

In order to investigate how purely monetary shocks can cause fluctuations in real variables, let us assume that the shock to the nominal interest rate $v$ in the Taylor rule follows a first order autoregressive process of the form,

\(^7\) See Mathematical Annex 5 or Blanchard and Kahn (1980).

\(^8\) See Bullard and Mitra (2002).
A positive value for $\varepsilon^v$ is a contractionary monetary shock, which leads to a rise in nominal interest rates for given inflation and excess output. A negative value is an expansionary monetary shock, leading to a fall in nominal interest rates for given inflation and excess output.

Solving the model in (14.43), or alternatively the three equations (14.40)-(14.42), under the assumption that there are only monetary and no real shocks, fluctuations of excess output and inflation are determined by,

$$y_t = -(1 - \beta \rho_v) \Lambda_v v_t = \rho_v y_{t-1} - (1 - \beta \rho_v) \Lambda_v \varepsilon^v_t \tag{14.46}$$

$$\pi_t = -\kappa \Lambda_v v_t = \rho_v \pi_{t-1} - \kappa \Lambda_v \varepsilon^v_t \tag{14.47}$$

where,

$$\Lambda_v = \frac{1}{(1 - \beta \rho_v) \left( \theta (1 - \rho_v) + \eta_v \right) + \kappa (\eta_\pi - \rho_v)} > 0 \tag{14.48}$$

One can easily show that $\Lambda_v$ is positive to the extent that (14.44) is satisfied. As a result, an exogenous increase to the nominal interest rate leads to a fall in both excess output and inflation, while an exogenous reduction in the nominal interest rate leads to a rise in excess output and inflation. Furthermore, to the extent that the shock is persistent, the effects on output and inflation persist, due to the existence of staggered pricing. If the shock in non-persistent, then the effects on excess output and inflation are non persistent as well.

Note that in the new keynesian model with predetermined periodic wage contracts, analyzed in Chapter 13, even persistent monetary shocks have non persistent effects, unless there is endogenous persistence in employment and output.

Obviously, monetary shocks are not the only ones causing aggregate fluctuations in the "new keynesian" model with staggered pricing. Real shocks cause aggregate fluctuations too.

### 14.2.2 Implications of a Real Productivity Shock

We have already demonstrated that real shocks, such as shocks to productivity $a$, cause fluctuations of the "natural" rate of real variables. However, due to staggered pricing, the evolution of real variables deviates from the evolution of their "natural" rates. As a result, given the monetary policy rule (14.42), real shocks cause fluctuations in excess output and inflation.

In order to investigate such fluctuations, let us assume that the shock to productivity follows an first order autoregressive process of the form,

$$a_t = \rho_a a_{t-1} + \varepsilon^a_t \tag{14.49}$$
where, $0 < \rho_a < 1$, and $\varepsilon_t^a \sim N(0, \sigma_a^2)$.

In contrast to pure monetary shocks, real shocks affect the evolution of the “natural” rate of real variables, such as the real interest rate $r^N$.

Solving the model using (14.49), and ignoring monetary shocks, we find that fluctuations in excess output and inflation are determined by,

\begin{align}
\hat{y}_t &= -\theta \psi (1 - \rho_a)(1 - \beta \rho_a) \Lambda_a \varepsilon_t^a = \rho_a \hat{y}_{t-1} - \theta \psi (1 - \rho_a)(1 - \beta \rho_a) \Lambda_a \varepsilon_t^a \quad (14.50) \\
\pi_t &= -\theta \psi (1 - \rho_a) \kappa \Lambda_a \varepsilon_t^a = \rho_a \pi_{t-1} - \theta \psi (1 - \rho_a) \kappa \Lambda_a \varepsilon_t^a \quad (14.51)
\end{align}

where,

$$
\Lambda_a = \frac{1}{(1 - \beta \rho_a)(\theta (1 - \rho_a) + \eta) + \kappa (\eta - \rho_a)} > 0 \quad (14.52)
$$

From (14.50), and under the assumption that the autoregressive coefficient of the productivity shocks is less than one, a positive productivity shock leads to a fall in excess output and inflation. This is because real output rises by less than its “natural rate”, due to the behavior of real interest rates. To the extent that productivity shocks are persistent excess output and inflation display persistent fluctuations in response to productivity shocks.

It is worth noting that there is no endogenous persistence in this model. The persistence of excess output and inflation arises solely because of the persistence on real and monetary shocks. However, because of staggered pricing, this persistence feeds through both real and nominal variables. In the case of the periodic wage setting model of Chapter 13, it is only innovations in nominal and real shocks that affect fluctuations in real variables, as it is only unanticipated inflation that generates real effects.

14.2.3 A Dynamic Simulation of the Model

It is worth simulating the model for particular parameter values, in order to assess its quantitative properties.

Figures 14.1 and 14.2 present the dynamic effects of both monetary and real shocks, for values of the parameters commonly used in the literature.

In Figure 14.1 we present the dynamic effects of a 0.25 basis points shock $\varepsilon^\nu$ in nominal interest rates. This shock leads to an automatic increase of the nominal and the real interest rate and reduces excess production and inflation. Because this shock does not affect the "natural" level of output, real output, employment and real wages decline. The economy gradually returns to long-run equilibrium, as the effects of the monetary shock gradually peter out.

In Figure 14.2 we present the dynamic effects of a 0.25 shock $\varepsilon^a$ to productivity. This shock leads to a prolonged rise of the “natural” level of output, a reduction of excess output and inflation, an
increase in the real wage, and a decline in nominal and real interest rates. The reduction in real interest rates leads to an increase in actual output, which however is smaller than the increase of the “natural” level of output. That is the reason that excess output falls, production decreases. Again, the economy gradually returns to equilibrium long as the effects of the actual disturbance progressively peter out.

### 14.3 Conclusions

In this chapter we have analyzed the structure and the properties of a “new keynesian” model, based on monopolistic competition and staggered pricing. Unlike traditional Keynesian models, in which the basic relations are not derived explicitly from microeconomic foundations, this new Keynesian model is a dynamic stochastic general equilibrium model based on explicit microeconomic foundations, analogous to those of “new classical” models.

After we presented the properties of this model, we analyzed the effects of monetary and real shocks to fluctuations in excess output and inflation.

This “new keynesian” model, unlike “new classical” models, can explain aggregate fluctuations caused by monetary and aggregate demand shocks. These shocks are transmitted to real variables and persist over time through staggered pricing. Such monetary cycles cannot result from models with immediate adjustment of wages and prices.

As in the fully competitive “new classical” models, even under monopolistic competition, when there is full adjustment of prices and wages, monetary shocks affect only nominal and not real variables such as output, consumption, employment, real wages and real interest rates. If there is partial stickiness of wages and prices, as in the “periodic nominal wage contracts” model of Chapter 13, or the “monopolistic competition and staggered pricing” model of this Chapter, then monetary shocks have real effects and monetary cycles can arise.

It is worth noting that whereas the model of Chapter 13, with real and nominal labor market distortions could account for “involuntary” unemployment, and fluctuations in the unemployment rate, in the model of this Chapter there is no involuntary unemployment. Output deviates from its “natural rate”, because of staggered pricing, but fluctuations in employment are due to inter-temporal substitution, and there is no involuntary unemployment, since the labor market is assumed fully competitive. This is a significant weakness of this particular model. This weakness can be addressed if one were to combine the two models, and rely on a “generalized new keynesian” model, with both staggered pricing, predetermined nominal wage contracts and real product and labor market distortions.

In the next chapter we delve deeper into the labor market distortions that can account for involuntary unemployment, and present the main elements of an important class of matching models of the labor market.
Annex to Chapter 14
The Rotemberg Model of Convex Costs of Price Adjustment

An alternative model of sluggish price adjustment is the Rotemberg (1982 a,b) model of costly price changes.

For the representative monopolistically competitive firm, as the one examined in section 14.1.2, the optimal price is given by,

\[ P^*_t = \frac{\varepsilon}{\varepsilon - 1} \left( \frac{W_t}{(1 - \alpha)A_tL_t^{-\alpha}} \right) \]  

(A14.1)

The optimal price is a constant markup on marginal costs. Marginal costs are equal to wage costs over the marginal productivity of labor. Note that because of decreasing returns to employment, increasing employment and output implies declining marginal productivity of labor and increasing marginal costs of production. Using the production function to substitute out for labor, (A14.1) can also be expressed as,

\[ P^*_t = \frac{\varepsilon}{\varepsilon - 1} \left( \frac{W_t(Y_t)^{\alpha/(1-\alpha)}}{(1 - \alpha)(A_t)^{1/(1-\alpha)}} \right) \]  

(A14.1′)

An increase in output increases the marginal costs of production for given wages, because of the declining marginal productivity of labor. Hence, with higher output the optimal price must rise.

In logs, (A14.1) and (A14.1′) imply,

\[ \ln P^*_t = \mu + w_t - a_t + \alpha l_t = \mu + w_t + \frac{1}{1 - \alpha} (\alpha y_t - a_t) \]  

(A14.2)

where,

\[ a_t = \ln A_t, \quad \mu = \ln \left( \frac{\varepsilon}{\varepsilon - 1} \right) - \ln(1 - \alpha). \]

\(a\) is the logarithm of the exogenous productivity shock, and the constant \(\mu\) is the logarithm of the markup on marginal cost, minus the logarithm of the coefficient implying decreasing returns to labor.

All firms, are assumed to be facing convex costs of adjusting prices. Rotemberg (1982, 1983) assumes that they balance the costs of deviating from their optimal price against the costs of adjusting prices. In the model that follows, following Rotemberg, we assume that firms set current prices minimizing a quadratic cost function which penalizes percentage deviations of prices from the optimal price, and the adjustment of prices from period to period. This takes the form,
where \( p \) is the log of the actual price of the representative firm. \( \xi \) is a parameter measuring the cost of adjustment of prices relative to the cost of deviations from the optimal price.

From the first order conditions for the minimization of (A14.3), it follows that,

\[
\begin{align*}
    p_t &= \frac{1}{1 + \xi(1 + \beta)} p_t^* + \frac{\xi}{1 + \xi(1 + \beta)} p_{t-1} + \frac{\xi \beta}{1 + \xi(1 + \beta)} E_t p_{t+1}
\end{align*}
\]  

(A14.4)

The current price, in logs, is a weighted average of the optimal price, the past price and the expected future price. The firm is forward looking, and anticipates the future costs of adjusting prices, so its current price depends not only on its past price, but on its expected future price as well. Since this is the representative firm, we can take in to be equal to the log of the price level.

Expressing (A14.4) as an inflation equation, one gets,

\[
\begin{align*}
    \pi_t &= \beta E_t \pi_{t+1} + \frac{1}{\xi} \left( p_t^* - p_t \right)
\end{align*}
\]  

(A14.5)

where \( \pi_t = p_t - p_{t-1} \) is the rate of inflation.

Inflation deviates from expected future inflation, to the extent that the optimal price exceeds the current price. Substituting for the optimal price from (A14.2), one gets,

\[
\begin{align*}
    \pi_t &= \beta E_t \pi_{t+1} + \frac{1}{\xi} \left( \mu + w_t - a_t + \alpha l_t - p_t \right)
\end{align*}
\]  

(A14.6)

From (A14.6), inflation deviates from expected future inflation, to the extent that the marginal cost of production plus the optimal price markup exceeds the current price. Using the labor and product market equilibrium conditions to substitute out for the real wage and employment, as well as the definition of the “natural rate” of output, we can express (A14.6) as,

\[
\begin{align*}
    \pi_t &= \beta E_t \pi_{t+1} + \kappa \left( y_t - y_t^\nu \right)
\end{align*}
\]  

(A14.7)

where \( \kappa = \frac{\theta(1-\alpha) + \alpha + \lambda}{\xi(1-\alpha)} > 0 \)

(A14.7) has exactly the same form as the “new keynesian” Phillips curve (14.40) derived from the Calvo (1983) model of staggered pricing. The only difference is in the definition of \( \kappa \) which is now in terms of the parameter \( \xi \) of the Rotemberg model, instead of the parameter \( \gamma \) of the Calvo model. Thus, the two models of sluggish price adjustment, the Rotemberg model of costs of adjustment of prices and the Calvo model of staggered pricing are observationally equivalent at the aggregate level.
References


Figure 14.1
The Dynamic Behavior of the Staggered Pricing Model
Following a Contractionary Monetary Shock

Note: The parameter values for this simulation are: $\theta=1$, $\lambda=1$, $\rho=0.01$, $\alpha=0.333$, $\varepsilon=6$, $\gamma=0.667$, $\eta_\pi=1.50$, $\eta_y=0.125$, $\rho_a=0.90$, $\rho_v=0.50$. 
Figure 14.2
The Dynamic Behavior of the Staggered Pricing Model Following a Productivity Shock

Note: The parameter values for this simulation are: \( \theta = 1, \lambda = 1, \rho = 0.01, \alpha = 0.333, \varepsilon = 6, \gamma = 0.667, \eta_e = 1.50, \eta_y = 0.125, \rho_a = 0.90, \rho_v = 0.50. \)